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KIMBER'S CONSTRUCTIONS

KEY TO MATRICULATION COURSE.

PRICE 5/-

A

KEY

TO THE

MATRICULATION COURSE OF MATHEMATICS

FOR THE

UNIVERSITY OF LONDON.

THIRD EDITION, REVISED AND EXTENDED.

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181. e. 50.

NOTE TO THE NEW AND REVISED EDITION.

In the First Edition of the "Key," many solutions of the early papers, and of important questions, were given entire; but these having been removed when the "Matriculation" and "B.A." Courses were revised, were made use of as Examples in those works to illustrate the principles and rules there established.

The present "Key," therefore, is not quite independent of the "Matriculation Course" with which it is connected, but refers constantly to Articles and Examples therein for explanations and full solutions.

Numerous corrections have been made in this Edition, which were partly supplied by the kindness of Correspondents who had used the book. To their valuable communications and polite attention I am much indebted, and to them my best thanks are cordially tendered.

The Solutions from 1865 to 1873 are now also added.

Corrections are still solicited, and will be thankfully received and carefully noted for use in future editions.

THOMAS KIMBER.

HABERDASHERS' SCHOOL, HOXTON, LONDON, N.
August 1873.

The "Key" has been again revised, and the Solutions extended to 1875 inclusive.

LONDON: *March* 1876.

A KEY

TO THE

MATRICULATION COURSE OF MATHEMATICS

FOR THE

UNIVERSITY OF LONDON.

1838. *November 5th.*

1. (1) Art. 49. (2) Art. 67. (3) P. 30, Ex. 1.
2. (1) P. 30, Ex. 2. (2) $5\frac{297}{427}$.
3. P. 44, Ex. 1.
4. (1) Art. 91. (2) 48·0384. (3) 170790.
5. 97531.
6. £133 18*s.* $4\frac{1}{4}$*d.*

7. (1) If $a\,b$ mean b taken a times, $b\,a$ must mean a taken b times. Suppose that the operations here indicated are performed in the simplest manner. First, let b things be placed in a horizontal line, and let this be repeated a times; we shall then have a lines, each containing b things, or b things taken a times, or $a\,b$ things. Now if this system of horizontal lines be arranged vertically, we shall then have b horizontal lines, each of which will contain a things, or a things taken b times, or $a\,b$ things, as above.

(2) Arts. 23—25.

8. P. 11, Ex. 1.
9. (1) P. 23, Ex. 1. (2) P. 23, Ex. 2.

$$(3)\ \frac{3\,x+9}{x^4-x^3+6\,x^2-6\,x+6}.$$

10. (1) $7+8\,x+x^2=x^2+8\,x+15$,

$\therefore 7=15$, or the equation is impossible.

(2) $x=4$. (3) $x=1$. (4) $x=1$.

11. Art. 136.
12. (1) *Art. 175, Ex. 6. $x=4$ or 1, $y=1$ or 4.
    (2) *Art. 175, Ex. 2. $x=1$, $y=1$.
    (3) *Arts. 185—8. $x=13$ or 5; $y=12$ or 4.
13. Arts. 102; 103; 145; 147.
14. (1) P. 59, Ex. 4. (2) P. 63, Ex. 11.
15. P. 48, Ex. 15.

1838. *November 7th.*

9. P. 46, Ex. 1.
10. Art. 89. $\frac{175}{9999}$.

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* These solutions are unavoidably given in the second part of the work; they are, however, almost the only instances which occur.

11. Art. 82.

12. Art. 154, Ex. 4.

13. Art. 41.

14. Art. 115. (Compare Ex. 3.) *Ans.* 100; or $-$ 100. (The signification of the negative quantity is explained in Art. 186.) See note on preceding page.

15. (1) $\sqrt{a} + \sqrt{b} + \sqrt{c} = \sqrt{a} + (\sqrt{b} + \sqrt{c})$.

1st. Multiply by $\sqrt{a} - (\sqrt{b} + \sqrt{c})$,

giving $a - (\sqrt{b} + \sqrt{c})^2$ or $a - b - c - 2\sqrt{bc}$.

2nd. Multiply by $a - b - c + 2\sqrt{bc}$,

giving $(a - b - c)^2 - 4bc$.

∴ the required multiplier is $(a - b - c + 2\sqrt{bc})(\sqrt{a} - \sqrt{b} - \sqrt{c})$.

(2) $2 + \sqrt{5} + \sqrt{20} = 2 + \sqrt{5} + 2\sqrt{5} = 2 + 3\sqrt{5}$,

and $1 + \sqrt{45} = 1 + 3\sqrt{5}$;

$$\text{Then } \frac{2 + 3\sqrt{5}}{1 + 3\sqrt{5}} = \frac{(2 + 3\sqrt{5})(1 - 3\sqrt{5})}{(1 + 3\sqrt{5})(1 - 3\sqrt{5})} = \frac{2 - 45 - 3\sqrt{5}}{1 - 45}$$

$$= \frac{-43 - 3\sqrt{5}}{-44}, \text{ or } \frac{43 + 3\sqrt{5}}{44} = 1\cdot129. \textit{ Ans.}$$

16. Let x be the number of sides; then, *Eucl.* I., 32, Cor. 2, the external angles = four right angles; also in a polygon of m sides, and of n sides, in each case the exterior angles = four right angles; ∴ the number of sides $= \frac{m + n}{2}$.

1839. *Monday, Oct. 7th.*

1. P. 30, Ex. 3.
2. (1) Art. 83. (2) Art. 83. (3) Art. 88.
3. (1) 127. (2) ·0123. (3) Arts. 95, 98.
4. Art. 156, ¶ 2, Ex. 1.
5. Arts. 23—6.
6. (1) Art. 29, Ex. 2. (2) Art. 36, Ex. 3.
7. (1) $\frac{7}{9}$. (2) $x - 3$.
8. (1) $x = \frac{2}{3}$. (2) $x = 1$. (3) P. 73, Ex. 2.
9. P. 59, Ex. 2.
10. Art. 146.
11. P. 63, Ex. 12.
12. Art. 37, ¶ 1, 4, 7.
13. $3234 = 11 \times 294 = 11 \times 6 \times 49$.

∴ to make 3234 a perfect square, it must be multiplied by $\frac{1}{66}$ or by 66.

1839. *Wednesday, Oct. 9th.*

11. (1) $x = 56$. (3) P. 67, Ex. 6.
(2) P. 67, Ex. 5. (4) „ „ Ex. 7.

12. (1) P. 71, Ex. 2. (2) P. 73, Ex. 3. (3) P. 73, Ex. 4.

13. Let a, b, c, d, be the shares in an adventure, and e the given result (gain or loss as the case may be); and put $a + b + c + d = S$, and let the share of the

Result due to a be w. $\therefore\ S : e :: a : w \quad \therefore\ w = \frac{a\,e}{S}$.
„ „ b „ x. $S : e :: b : x \quad x = \frac{b\,e}{S}$.
„ „ c „ y. $S : e :: c : y \quad y = \frac{c\,e}{S}$.
„ „ d „ z. $S : e :: d : z \quad z = \frac{d\,e}{S}$.

1840. *October 5th.*

1. Arts. 71, 72.
2. (1) 321. (2) ·2753 · · · · (3) Arts. 95, 98.
3. 1 cwt. : 275 cwt. 3 qrs. 25 lbs. : : £3 7*s.* $6\frac{3}{4}$*d.*
: £932 5*s.* $5\frac{1}{4}\frac{15}{112}$ *Ans.*
4. P. 87, Ex. at top.
5. P. 36, Art. 82. See 1838, Nov. 7, Question 11.
6.

$$\frac{2\sqrt{18}+\sqrt{32}}{\sqrt{56}+3\sqrt{14}} = \frac{2\sqrt{9\times 2}+\sqrt{16\times 2}}{\sqrt{4\times 14}+3\sqrt{14}} = \frac{6\sqrt{2}+4\sqrt{2}}{2\sqrt{14}+3\sqrt{14}}$$
$$= \frac{10\sqrt{2}}{5\sqrt{14}} = 2\sqrt{\tfrac{1}{7}} = 2\sqrt{\cdot\dot{1}4285\dot{7}} = \cdot 755 \text{ nearly.} \quad \textit{Ans.}$$

7. (1) P. 21, Arts. 42, 46. (2) $\frac{x^2 - 5\,x\,y + 4\,y^2}{x - 3\,y}$ *Ans.*
8. P. 66, Art. 131, Exs. 1, 3, 2.
9. (1) $x = 25\frac{4}{11}$, $y = 34\frac{6}{11}$. (2) $x = 5$, $y = 7$, $z = 15$.
10. 30 hours. *Ans.*
11. P. 53, Arts. 101 and 102.
12. P. 59, Ex. 3.
13. From the first condition $x = 17\,p + 7$, where p is the quotient and 7 the remainder, in dividing the number by 17.

From the second condition $17\,p + 7 = 26\,q + 13$, where q is the quotient and 13 the remainder, in dividing the number by 26;

$$\therefore\ p = \frac{26\,q + 6}{17},$$

where we may give q any value that makes p an integer.

$$\frac{26\,q + 6}{17} = q + \frac{9\,q + 6}{17},$$

where we have to make $\frac{9\,q + 6}{17}$ or $\frac{3\,q + 2}{17}$ the least possible integer.

But this is the case when $q = 5$, $\therefore$ if $q = 5$, $26\,q + 13 = 143$, the number required.

1840. *Thursday, October 8th.*

9. $x^2 - 10\,x + 25$.

10. Let the time be x hours, then

The work done by the 1st in one hour is $\frac{1}{a}$.

" " 2nd " $\frac{1}{b}$.

" " by both " $\frac{1}{x}$.

$$\therefore \frac{1}{a} + \frac{1}{b} = \frac{1}{x},$$

$$a\,x + b\,x = a\,b \quad \therefore x = \frac{a\,b}{a + b}.$$

11. (1) $x = 17$. (2) $x = 10$. (3) P. 66, Art. 131, Ex 4.

12. (1) is impossible. (2) and (3) are identical.

13. (1) $x = 10, y = 27$. (2) $x = 6, y = 7, z = 8$. (3) $x = 4$, $y = 3$, $z = 5$.

1841. *October 4th.*

1. (1) P. 37, Art. 83. (2) See Arts. 90, 92.
$6{\cdot}012 \times {\cdot}00345 = {\cdot}0207414$.
$6{\cdot}012 \div {\cdot}00345 = 1742{\cdot}\dot{6}08695652173913043478\dot{2}$.

2. £723 4*s*. 5¼*d*. $\frac{3}{5}$.

3. P. 57, Ex. 2.

4. (1) 2121. (2) ·2121.

5. (1) See P. 11, Art. 27. (2) $x^3 + 3\,x^2\,y + 3\,x\,y^2 + y^3$. (3) $1 - 4\,x^2 + x^3 + 4\,x^4$.

6. (1) P. 13, Arts. 31 to 36. (2) $x^2 - 5\,x + 6$. (3) P. 15, Ex. 1.

7. (1) P. 79, Art. 145. (2) P. 82, Ex. 2.

8. (1) P. 60, Arts. 117, 118. (2) P. 59, Ex. 6.

9. (1) $x = 11$. (2) $x = 60$. (3) $x = 2\frac{1}{47}$.
(4) $x = \frac{415}{522}$, $y = \frac{206}{261}$. (5) $x = -34$. (6) is identical.

10. Let x = number of hare-leaps; $2 : 3 :: 300 : x$ $\therefore x = 450$, the hare was 50 leaps in advance, so that 300 greyhound-leaps = 500 hare-leaps; and 300 : 500 :: 1 : $1\frac{2}{3}$, or the greyhound's leap contains $1\frac{2}{3}$ of the hare's.

11. Assuming that $r > 1$, let k be any whole number greater than r, and let $\frac{k}{r} = p + \frac{a_0}{r}$; $\therefore k = p\,r + a_0$. If p be not $< r$, let $\frac{p}{r} = q + \frac{a_1}{r}$; $\therefore p = q\,r + a_1$, $\therefore k = q\,r^2 + a_1\,r + a_0$; and pro-

ceeding thus till the quotient is less than r, we shall have the form proposed in the question.

1841. *October 6th.*

8. (1) $x = 532$. (2) $x = 120$. (3) $x = 41$. (4) Art. 131, Ex. 8.

$$(5)\ x = \frac{s+d}{2}, \quad y = \frac{s-d}{2} \qquad (6)\ \left.\begin{array}{l} x = 115 \\ y = 5 \end{array}\right\}.$$

9. P. 77, Ex. 10.

1842. *Monday, July 4th.*

1. Arts. 1 to 3.
2. Arts. 34 and 78.
3. (1) Art. 70, Ex. 4, p. 31. (2) Art. 70, Ex. 5, p. 31. (3) ·00357.
4. Arts. 95 . . . . 98. (1) 1842. (2) 1·93649.
5. (1) P. 88, Art. 159. (2) Ex. 1, p. 89.
6. P. 10, Art. 25. (1) Ex. 3, p. 12. (2) $x^6 - 3\,a^2\,x^4 + 3\,a^4\,x^2 - a^6$.

(3) Here $(x+1)^5 + (x-1)^5$ by actual expansion and addition are, $(x^5 + 5\,x^4 + 10\,x^3 + 10\,x^2 + 5\,x + 1) + (x^5 - 5\,x^4 + 10\,x^3 - 10\,x^2 + 5\,x - 1)$

$$= 2\,x^5 + 20\,x^3 + 10\,x,$$

$$\therefore\ x^5 = \frac{2\,x^5 + 20\,x^3 + 10\,x}{2} - (10\,x^3 + 5\,x);$$

$$\text{or } x^5 = x^5 + 10\,x^3 + 5\,x - 10\,x^3 - 5\,x;$$

$$\therefore\ x^5 = x^5.$$

7. (1) Ex. 2, p. 15. (2) P. 36, Art. 82, (β) and (3).
8. P. 82, Ex. 3, and p. 11 of this "Key," 1849, Ex. 10.
9. (1) $x = 11$. (2) $x = 54$. (3) $x = \dfrac{2\,a^2 - 3\,a\,b}{6\,(a-b)}$.
   (4) $x = 12, y = 10$. (5) $x = 17, y = 9, z = 1$.

1842. *Wednesday, July 6th.*

9. (1) $x = 1$. (2) $x = -\,6\frac{712}{5711}, y = 8\frac{465}{5711}, z = 1\frac{1831}{5711}$.
   (3) $x = -\,72\frac{7}{13}, y = 25$.
10. P. 75, Ex. 4.

1843. *Monday, July 3rd.*

1. (1) Art. 22, p. 10. (2) Art. 31, p. 13.
2. (1) P. 25, Arts. 49, 52. (2) Art. 81, Ex. 1, p. 35.
3. $\frac{60}{120}, \frac{80}{120}, \frac{72}{120}, \frac{105}{120}, \frac{108}{120}, \frac{110}{120}, \frac{104}{120}$. (2) P. 29, Art. 67.
   (3) $\frac{11}{12}$.

4. (1) Ex. 2, p. 46. (2) $\frac{1}{6}$.

5. (1) $1\frac{1}{4}$. (2) $\frac{1}{2500}$. See Art. 83, p. 37. (3) ·00011.

6. (1) 345. (2) 1·1. (3) 3·47850542 . . . .

7. £433 4*s.* $7\frac{3}{4}$*d.*

8. (1) Art. 70, p. 31, Ex. 6. (2) $x^3 - a\,x^2 - a^2 x + a^3$.

(3) $x - 4$. (4) $x^{\frac{2}{3}} + x^{\frac{1}{3}}\,y^{\frac{1}{3}} + y^{\frac{2}{3}}$.

9. P. 53, Art. 101. The rule of three. P. 79, Art. 142.

10. (1) $x = 3$. (2) $x = 12$. (3) $x = 10$. (4) $\left.\begin{array}{r} x = 6 \\ y = 3 \end{array}\right\}$.

11. Ex. 5, p. 75.

1843. *Wednesday, July 5th.*

9. (1) $x = 9$. (2) $x = 7, y = 17$. (3) Art. 121, Ex. 7.

1844. *Monday, July 1st.*

1. (1) Arts. 1 to 3. (2) $4765 \times 205 = 976825$. (3) P. 9, Arts. 21, 22.

2. (1) P. 26, Art. 55. (2) $\frac{5}{6}$.

3. (1) P. 46, Ex. 3. Again, £$\frac{1}{19}$ = 1*s.* $0\frac{1}{2}$ $\frac{10}{19}$*d.* Gui. $\frac{1}{20}$
1*s.* $0\frac{1}{2}$ $\frac{8}{20}$*d.* and the diff. is $\frac{48}{380}$ of a farthing, or $\frac{1}{380}$ of a shilling.
(2) Ex. 4, p. 46.

4. £714.

5. (1) and (2) P. 37, Art. 83. (3) ·0096. (4) P. 39, Art. 87.
(5) Art. 88.

6. Art. 95. (2) 12·5. (3) ·08. (4) ·2529822 . . . &c.

7. P. 49, Ques. 5, p. 91.

8. (1) $\frac{41\,a - 26\,b}{28}$. (2) $x - y$. (3) $x^5 - 32\,a^5$. (4) $4\,a^2 - 7\,b^2$.

(5) $x^{\frac{4}{5}} + x^{\frac{3}{5}}\,y^{\frac{1}{5}} + x^{\frac{2}{5}}\,y^{\frac{2}{5}} + x^{\frac{1}{5}}\,y^{\frac{3}{5}} + y^{\frac{4}{5}} + \frac{2\,y}{x^{\frac{1}{5}} - y^{\frac{1}{5}}}$.

9. (1) $x = 12$. (2) $x = 5$. (3) $x = 4$. (4) $\left.\begin{array}{r} x = 6 \\ y = 7 \end{array}\right\}$.

10. (1) P. 53, Art. 101. (2) See Art. 147, p. 80. (3) 7.

11. Ex. 6, p. 76.

1845. *Monday, July 7th.*

1. (1) £25932 13*s.* $3\frac{1}{2}$*d.* (2) 15. Arts. 30 and 31, p. 13.

2. Art. 67, p. 29. The given fractions are $\frac{70}{105}, \frac{84}{105}, \frac{90}{105}$.

(2) The *least* com. denom. (Art. 68) of the given fractions is 1260.

3. (1) $\frac{181}{480}$. (2) £405 10*s.* 5*d.* (3) $16992553\frac{1}{13}$ seconds.

4. In agriculture 7·95 per cent.: in trade and manufacture 16·5 per cent.

5. (1) Art. 83, p. 37. (2) ·017359. (3) ·0005. (4) Art. 92, p. 43.

$$(5)\ \frac{2\cdot5}{\cdot00125} = 2000.$$

6. Arts. 59, p. 27, and 85, p. 39.

7. (1) The answer is only an approximation, more and more nearly true the further it is carried, but never quite correct.

(2) 126. (3) ·11.

(4) $\frac{12}{13}$. (5) $\sqrt[4]{2\cdot0736} = \sqrt{1\cdot44} = 1\cdot2$.

8. P. 31, Ex. 7. (1) xy. (2) $x^3 - 4x^2 - 17x + 60$.

(3) $x^5 + 2x^4 + 4x^3 + 8x^2 + 16x + 32$. (4) $x - y$.

9. (1) $x = 60$. (2) $x = 14$. (3) $x = 5$. (4) $x = 144$, $y = 216$.

10. P. 60, Ex. 8. (1) 2550. (2) -296. (3) $14\frac{2}{5}$.

11. P. 82, Ex. 6.

1846. *Monday, July 6th.*

1. £36 0*s.* 5*d.* Read and apply Arts. 11 and 94.

2. (1) Arts. 49, 51, p. 25. (2) Art. 55, p. 26. (3) $\frac{403}{899} = \frac{13}{29}$.

3. P. 16, Art. 37, ¶ 4.

4. (1) The 3 per cent. is the more productive by 1*s.* 8*d.* per cent. (2) £51 4*s.* 0½*d.*

5. P. 37, Art. 83. (1) is equivalent to multiplying by 100. (2) is equivalent to dividing by 100. (3) $\frac{3}{4}$, $\frac{1}{2000}$.

6. (1) P. 40, Art. 88. (2) $\cdot\dot{7}1428\dot{5}$.

7. (1) 3·51. (2) Art. 95, p. 48.

8. (1) Art. 7. (2) Ex. 8, p. 31. (3) Ex. 9, p. 31.

(4) $a^3 - 5a^2x + 10ax^2 - 8x^3$. (5) $4x^2 - 3x - 2$.

$$(6)\ x^{\frac{4}{5}} + x^{\frac{3}{5}}y^{\frac{1}{5}} + x^{\frac{2}{5}}y^{\frac{2}{5}} + x^{\frac{1}{5}}y^{\frac{3}{5}} + y^{\frac{4}{5}}$$

9. (1) $x = 12$. (2) $x = 40$. (3) $x = 1$. (4) $x = 2, y = 1$.

10. (1) P. 53, Arts. 100 and 101. When three quantities, a, b, c, are in continued proportion, the first has to the third the *duplicate* ratio of the first to the second, or if $a : b :: b : c$, then $a : c :: a^2 : b^2$. (2) P. 83, Art. 152. (3) Multiply extremes and means together, as shown in Art. 102, then dividing by $b\,d$, $\frac{a}{d} = \frac{b}{c}$. (4) P. 80, Art. 147.

11. Ex. 7, p. 76.

1846. *Wednesday, July 8th.*

12. P. 60, Ex. 9. (1) 8100. (2) 8190. (3) 1023. (4) 5. (5) P. 59, Exs. 4 and 5. (6) The sum of an indefinite series is the limit to which we continually approach by taking more and more terms of the series, but which cannot be reached by any finite number of terms.

1847. *Monday, July 5th.*

1. (1) 271 yds. 1 ft. 7 in. (2) 2467, the quotient; true value of first remainder 1611000.
2. P. 55, Art. 109 (1).
3. (1) Art. 40, p. 20. (2) 19. (3) Art. 41, p. 20.
4. (1) P. 34, Arts. 76, 80, and 81. (2) Art. 75, p. 33.
5. (1) P. 88, Art. 159 and note. (2) Interest £196 12*s.* $6\frac{1}{2}$*d.* $\frac{2}{5}$. Discount £173 4*s.* $9\frac{1}{2}$*d.* $\frac{110}{227}$.
6. (1) $\frac{1}{2}$, $\frac{37}{1000}$. (2) and (3) Read Art. 83, p. 37.
7. (1) P. 49, Art. 95. (2) 436. (3) ·08. (4) ·284604, &c.
8. (1) P. 8, Ex. 1. (2) $2a^2 + 2b^2 + 2c^2 - 4bc$. (3) P. 9, Ex. 5. (4) $a^4 - 2a^2b^2 + b^4$. (5) P. 12, Ex. 5. (6) $x - 20$. (7) $\frac{x}{x-3}$.
9. (1) $x = 4\frac{2}{7}$. (2) $x = 12$. (3) $x = 5, y = 6$. (4) $x = 9$.
10. Ex. 9, p. 77.
11. (1) Art. 112, p. 58. (2) Art. 116, p. 60. (3) Ex. 1, p. 59.

$$(4)\quad S = \left\{\frac{7}{2} - \frac{1}{2}(13)\right\} 14 = -42. \quad \textit{Ans.}$$

$$(5)\quad \frac{3}{64}, \frac{3}{32}, \frac{3}{16}, \frac{3}{8}, \frac{3}{4}, \frac{3}{2}, 3, 6, 12, 24, 48, 96, 192, 384, 768.$$

1848. *Tuesday, July 4th.*

1. (1) £43 1*s.* $0\frac{1}{2}$*d.* (2) 51623232. (3) P. 9, Arts. 21 and 22.
2. P. 46, Ex. 6.
3. (1) P. 16, Art. 37, ¶ 3. (2) P. 21, Ex. 2.
4. (1) P. 26, Arts. 55, 67. (2) P. 29, Art. 68.

$$(3)\quad \frac{54}{72}, \frac{60}{72}, \frac{63}{72}, \frac{40}{72}, \frac{66}{72}.$$

5. (1) Interest £371 2*s.* $3\frac{1}{2}$*d.* $\frac{2}{5}$. (2) Present value £1211 0*s.* $3\frac{39}{227}$*d.*
6. (1) ·013. (2) ·0304. (3) Art. 87, p. 39. (4) Art. 88, p. 40.
7. (1) 123. (2) 72·03. (3) Art. 95, p. 49.
8. (1) P. 8, Ex. 2, (2) P. 9, Ex. 6. (3) $x^4 - 6x^3 + 13x^2 - 19x + 21$. (4) P. 13, Ex. 6. (5) $x^2 + 2x - 1$.

9. (1) $x = 26$. (2) $x = 5$. (3) $x = 8, y = 6$. (4) P. 72, Ex. 1.

10. (1) Art. 102. (2) If the four magnitudes be four straight lines, the rectangle under the extremes is equal to the rectangle under the means. See *Euclid*, VI. 16, and *Mat. Exam.* 1845, Quest. 11. (3) P. 82, Ex. 6 (2).

11. (1) Art. 114. (2) 1134. (3) -12. (4) 32.

1849. *Tuesday, July 3rd.*

1. (1) £5 6*s.* 4*d.* (2) 91000.

2. (1) $765468 = 700000 + 60000 + 5000 + 400 + 60 + 8$, or $7 \times 10^5 + 6 \times 10^4 + 5 \times 10^3 + 4 \times 10^2 + 6 \times 10 + 8 \times 10^0$. (2) Art. 37, ¶ 4.

3. (1) $3\frac{65}{72}$. (2) Art. 65. (3) $\frac{8}{11}$. Only one such fraction can be found, but it may be expressed in many equivalent forms.

4. P. 48, Ex. 14. (1) $\frac{9}{15800}$. (2) $\frac{81}{7900}$ of an inch.

5. (1) £6 11*s.* 6¾*d.* per cent. (2) £1338 16*s.* 9*d.*

6. (1) ·120508. (2) 13.

$$(3)\ 2{\cdot}564 = \frac{641}{250},\ {\cdot}047 = \frac{47}{1000};$$

$$2{\cdot}564 \times {\cdot}047 = \frac{641}{250} \times \frac{47}{1000} = \frac{30127}{250000} = {\cdot}120508.$$

$${\cdot}00169 = \frac{169}{100000},\ {\cdot}013 = \frac{13}{1000};$$

$$\frac{169}{100000} \div \frac{13}{1000} = \frac{13}{100} = {\cdot}13.$$

7. (1) ·0272. (2) Art. 88. (3) 508·3.

8. (1) $12\,a\,x + 2\,x\,y$. (2) $4\,a\,b$.

(3) $a^6 - 5\,a^5 x + 13\,a^4 x^2 - 24\,a^3 x^3 + 27\,a^2 x^4 - 23\,a\,x^5 + 3\,x^6$.

(4) $x - 2$. (5) $x^{-\frac{2}{3}} + a^{-\frac{1}{3}}\,x^{-\frac{1}{3}} + a^{-\frac{2}{3}}$.

9. (1) $x = 6\frac{1}{2}$. (2) $x = 8$. (3) $x = 8, y = 1$. (4) $x = 10, y = 8$.

10. (1) Art. 101. (2) 10800. (3) Art. 142.

If $a : b :: c : d :: e : f$, &c., then

$$a : b :: a + c + e + \&c. : b + d + f + \&c.$$

Let $\frac{a}{b} = r, \therefore \frac{c}{d} = r, \frac{e}{f} = r,$

$$\therefore a = b\,r,\ c = d\,r,\ e = f\,r, \ \&c.$$

$$\therefore a + c + e + \&c. = (b + d + f + \&c.)\,r,$$

$$\therefore \frac{a + c + e + \&c.}{b + d + f + \&c.} = r = \frac{a}{b},$$

$$\text{or } a : b :: a + c + e + \&c. : b + d + f + \&c.$$

11. (1) and (2) Arts. 112, 116. (3) $\frac{a+b}{2}$, $\sqrt{a\,b}$.

(4) Here $a = 7$; $n = 12$; $d = 5$; whence (Art. 113) $l = a + (n-1)\,d$ becomes $l = 7 + 11 \times 5 = 62$.

(5) Here $S = \frac{a\,r^n - a}{r-1}$ becomes $S = \frac{\frac{1}{3}\left(-\frac{2}{3}\right)^5 - \frac{1}{3}}{-\frac{2}{3} - 1}$,

$$= \frac{\frac{-32}{729} - \frac{243}{729}}{-\frac{5}{3}} = \frac{-\frac{275}{729}}{-\frac{5}{3}} = \frac{55}{243}.$$

1850. *July 2nd.*

1. £3,520,000.

2. (1) Those factors which have no divisors except unity, or which are prime numbers. See Art. 39. (2) Art. 37, ¶ 1, 4, 7. (3) 66.

3. (1) 972,658. (2) 14·142624, &c.

4. (1) Arts. 75 . . . 81. Exs. $\frac{77}{104}$, $1\frac{1}{3}$.

5. (1) Art. 83. (2) ·375. (3) ·000,000,375. (4) 356·144,685.
(5) 160,000.

6. (1) ·132,812,5. (2) $2^6 \times 5^x$ and $5^6 \times 2^x$, where x may have any of the values 0, 1, 2, 3, 4, 5, 6. See Art. 88, ¶ 1 and 2.

7. (1) Art. 95. (2) 34·56. (3) $\frac{12}{13}$. See solutions *M.* 1857. 5 [2].

8. (1) Art. 7. (2) Art. 7. (3) P. 9, Ex. 3. (4) P. 31, Ex. 10.
(5) $81\,x^4 - 21\,x^3\,y + 7\,x^2\,y^2 - y^4$. (6) $5\,x - 3\,y$.

9. (1) $x = 29$. (2) $x = 29$. (3) $x = 4$. (4) $x = 8$, $y = 7$.

10. (1) Art. 101. (2) That the product of the extremes equals the product of the means. See Arts. 101, 107. (3) 25 days. (4) Art. 145, and *M.* 1845, 11 (2).

11. (1) Arts. 112, 113, and 116.
(2) a, $a + 2$, $a + 4$, $a + 6$, &c., and a, $2\,a$, $4\,a$, $8\,a$, &c.
(3) $a + 38$ and $32\,a$.
(4) 15th term 219, and the sum = 1710. (5) P. 62, Ex. 8.

1851. *July 1st.*

1. 6632. Art. 11, Note.

2. Art. 37, ¶ 3, 4.

3. Arts. 155 and 159. Interest £477 19*s.* $4\frac{1}{5}$*d.*; discount £421 2*s.* $4\frac{4}{227}$*d.*

4. (1) Art. 67. (2) $\frac{300}{420}, \frac{385}{420}, \frac{390}{420}, \frac{392}{420}, \frac{357}{420}, \frac{380}{420}$. (3) $\frac{41}{456}$.

5. (1) Art. 83.

(2) $341{\cdot}7205 = 3 \times 10^2 + 4 \times 10^1 + 1 \times 10^0 + \frac{7}{10^1} + \frac{2}{10^2} + \frac{0}{10^3} + \frac{5}{10^4}$.

(3) Art. 89. (4) 2·712464. (5) 649·6. (6) 1,200,000.

6. (1) ·03616. (2) Art. 88. (3) Art. 87.

7. (1) 650·4. (2) When they have no common measure except unity, and see Art. 149.

(3) In $\sqrt{13}$ the process does not terminate, for if it did, we should have a fraction, which being multiplied by itself, would produce a whole number, and this is impossible.

8. (1) Art. 7. (2) $\frac{3\,a}{2} - \frac{b}{15} - \frac{c}{12}$. (3) P. 9, Ex. 7.

(4) $10\,x^5 - x^4\,y - 34\,x^3\,y^2 + 40\,x^2\,y^3 - 18\,x\,y^4 + 3\,y^5$.

(5) $3\,x^2 - 5\,x\,y + 2\,y^2$. (6) $\frac{2\,x^2 - x}{x^2 + x + 1}$.

9. (1) $x = 3$. (2) $x = 12$. (3) $x = 5,\ y = 2$. (4) $x = 5,\ y = 5$.

10. (1) Arts. 100, 101. (2) Art. 146. (3) Art. 145, and by inversion.
(4) Art. 148.

11. (1) Art. 112. (2) Art. 116.

(3) In arithmetical progression $b = \frac{a + c}{2}$, in geometrical progression $b = \sqrt{a\,c}$.

(4) 1330. (5) $\frac{2}{729}$. (6) $\frac{3}{8}$.

(7) P. 62, Ex. 9; and the three means are $\frac{7}{2}, \frac{49}{6}, \frac{343}{12}$.

1852. *July 6th.*

1. (1) £365 17*s.* 9½*d.* (2) Compare Arts. 31 and 94.

2. (1) Art. 40. (2) 105. (3) Art. 41.

3. (1) £145 5*s.* 2¼*d.*⅗. (2) £1 15*s.* (3) 8⅓ per cent.

4. (1) $\frac{273}{1150}$. (2) Arts. 75, 76. (3) $\left(\frac{2}{3}\right)^n < \frac{1}{8}$.

Let $n = 2, 3, 4$, &c., successively.

$$\text{Then} \left(\frac{2}{3}\right)^n = \left(\frac{2}{3}\right)^2 = \frac{4}{9} \text{ which is} > \frac{1}{8},$$

$$,, \quad \left(\frac{2}{3}\right)^n = \left(\frac{2}{3}\right)^3 = \frac{8}{27} \quad ,, \quad > \frac{1}{8},$$

$$,, \quad \left(\frac{2}{3}\right)^n = \left(\frac{2}{3}\right)^4 = \frac{16}{81} \quad ,, \quad > \frac{1}{8},$$

$$,, \quad \left(\frac{2}{3}\right)^n = \left(\frac{2}{3}\right)^5 = \frac{32}{243} \quad ,, \quad > \frac{1}{8},$$

$$,, \quad \left(\frac{2}{3}\right)^n = \left(\frac{2}{3}\right)^6 = \frac{64}{729} \quad ,, \quad < \frac{1}{8},$$

∴ 6 is the least value of n.

5. (1) $\frac{5}{10}, \frac{7}{100}, \frac{3}{1000}$. (2) To the *right.* Art. 83.
(3) Moving the point two places to the right. (4) ·000,000,7.
(5) 161·689,72. (6) ·0039. (7) 1,600,000.

6. (1) Art. 87. (2) $\cdot\dot{7}1428\dot{5}$. Art. 88, Note 2. (3) $\frac{5411}{6660}$.

$$(4)\ 3{\cdot}14159 - 3{\cdot}141 = {\cdot}00059 = \frac{59}{100000}.$$

$$3{\cdot}142 - 3{\cdot}14159 = {\cdot}00041 = \frac{41}{100000}.$$

In the former case we have a deficiency of $\frac{59}{100000}$, and in the latter an excess of $\frac{41}{100000}$. Therefore 3·142 is the nearer by $\frac{9}{50000}$.

7. (1) Art. 95. (2) [1] 5·67. [2] ·3794733.

8. (1) Art. 7. 1850, *M.* Quest. 8. (2) P. 9, Ex. 4.
(3) $4\,a\,b$. (4) $x^5 - 9\,x^4 + 23\,x^3 - 32\,x^2 + 26\,x - 9$.
(5) $x^3 + 3\,x^2\,y + 9\,x\,y^2 + 27\,y^3$.
(6) Art. 82. 1838, *M.* Nov. 7, Quest. 11.

9. (1) $x = 5$. (2) $x = 13$. (3) $x = 1\frac{1}{3}$. (4) $x = 10$, $y = 24$.

10. *a.* (1) P. 82, Ex. 3, and p. 11 of this "Key," 1849, Ex. 10.

(2) $\because c : d :: e : f, \therefore \frac{c}{e} = \frac{d}{f}$.

Subtract unity from each side.

$$\therefore \frac{c}{e} - 1 = \frac{d}{f} - 1, \text{ or } \frac{c-e}{e} = \frac{d-f}{f}.$$

$$\therefore \frac{c-e}{d-f} = \frac{e}{f} = \frac{a}{b}, \text{ by the question.}$$

$$\therefore a : b :: \overline{c-e} : \overline{d-f}.$$

β. (1) Art. 112.
(2) $a - 3\,b, a - 2\,b, a - b, a, a + b, a + 2\,b, a + 3\,b$.
(3) Last term 79, sum 820. (4) Art. 116.
(5) P. 63, Ex. 10.

1853. *July 5th.*

1. (1) The number of times the latter can be subtracted from the former.

(2) *Ans.* 704. (3) Art. 31.

2. (1) Art. 37, ¶ 6.

(2) Those prime numbers which multiplied together produce the given number.

(3) Art. 39, Ex. 1. 2, 3, 5, 7, 43, are the prime factors, and $54180 = 2^2 \times 3^2 \times 5 \times 7 \times 43$.

3. (1) £1851 19*s.* $3\frac{1}{4}$*d.* $\frac{1}{5}$. (2) P. 91, Ex. 6.

4. (1) Art. 65.

(2) $\dfrac{1820 + 1785 + 1900 + 1932 + 2030}{2100} = \dfrac{9467}{2100} = 4\frac{1067}{2100}.$

(3) $\overset{s.}{15} - \overset{s.}{4}\frac{1}{2} = \overset{s.}{10}\frac{1}{2}$, and $\dfrac{10\frac{1}{2}}{21} = \dfrac{1}{2}.$

(4) Compare solution, p. 82, Ex. 2.

5. (1) *Ans.* ·7, ·000013.

(2) Art. 83, Quest. 6, (2), (3), *Matric. Exam.*, 1847, July 5th.

(3) *Ans.* ·01825.

(4) A cipher should be interposed between the decimal point and the first digit.

6. (1) 33·022436. (2) 125·37513. (3) 67·58. (4) 50,000.

(5) 46·04. (6) $\dfrac{6{\cdot}25}{{\cdot}000125} = \dfrac{6250000}{125} = 50{,}000$ as in (4).

7. (1) and (2) Art. 88, and Note 2.

(3) Art. 89. [1] $\frac{71}{99}$. [2] $\dfrac{80574}{99900} = \dfrac{13429}{16650}$.

(4) $\cdot 3333\dot{3} = \frac{1}{3}$, and $\frac{1}{3}$ of $2\frac{1}{2}$ guineas = £0 17*s.* 6*d.*

8. (1) Art. 25. (2) $19\,x$. (3) $3\,a^2 + 16\,a\,b + 5\,b^2$.

(4) $a^6 - 4\,a^5 b + 7\,a^4 b^2 - 9\,a^3 b^3 + 10\,a^2 b^4 - 7\,a\,b^5 + 2\,b^6$.

(5) $x - 2$. (6) $x^3 - 6\,a\,x^2 + 12\,a^2\,x - 8\,a^3$.

9. (1) $x = 16$. (2) $x = 12$. (3) $x = 7$; $y = 6$.

(4) $x = 4$; $y = 5$; $z = 6$.

10. (1) Arts. 112 and 116. (2) 288. (3) 189. (4) $2\,a + d$.

(5) [1] Art. 101, [2] P. 82, Ex. 6 (2).

1854. *July 4th.*

1. (1) £1365 17*s.* 11*d.* × 63 = £86051 8*s.* 9*d.*

(2) P. 47, Ex. 7. 4 tons 13 cwt. 2 qrs. 15 lbs. 13 oz.

2. (1) Art. 37. [1] ¶ 4. [2] ¶ 7. (2)

$$\begin{array}{r} 3\underline{)2145} \\ 5\underline{)715} \\ 11\underline{)143} \\ 13 \end{array}$$

∴ all the *prime* divisors of 2145 are 3, 5, 11, 13.

3. (1) Art. 155. £3263 3*s.* $10\frac{3}{4}$*d.* $\frac{1}{5}$.

(2) Art. 158, ¶ IV. $4\frac{1}{2}$ per cent is paid.

4. (1) Arts. 49 and 55. (2) $\dfrac{31}{38}$.

(3) In Art. 45, it is proved that P the common measure of two numbers, Q and R, also measures their sum and difference. Now suppose P, the *g. c. m.* of Q and R, then since P *measures* $Q - R$, it cannot *exceed* their difference; therefore, the greatest common measure of two numbers cannot exceed the difference between them.

(4) $\frac{49}{30}$ of a crown. (5) $\frac{2}{5}$ of 12*s.* 6*d.*

5. $\frac{5}{10}$, $\frac{7}{10000}$, ·625, ·000012.

(1) 76. (2) 72·3265. (3) 31·75. (4) 130000. (5) 1·3.

(6) 3·79473 . . . . .

In example (2) the *true values* of the lines obtained in performing the multiplication are—

1st line = ·1139, 2nd line = 1·5946,

3rd do. = 2·278, 4th do. = 68·34.

6. (1) ·1808. (2) Art. 88. (3) P. 44, Ex. 3. (4) $\frac{24346}{27775}$.

(5) $\cdot 333\dot{3} = \frac{1}{3}$, $\cdot 2121\dot{2}\dot{1} = \frac{21}{99} = \frac{7}{33}$;

$$\text{and } \frac{1}{3} \times \frac{7}{33} = \frac{7}{99} = \cdot 0707\dot{0}\dot{7}.$$

7. (1) Art. 17.

(2) $(3\,a - 2\,b + 5\,c) - \{2\,a - 2\,b + 4\,c\}$

$$= 3\,a - 2\,b + 5\,c - 2\,a + 2\,b - 4\,c = a + c.$$

(3) $6\,x^5 - 11\,x^4\,y + 15\,x^3\,y^2 - 22\,x^2\,y^3 + 2\,y^5$.

(4) $\dfrac{a^4 - 4\,a^3\,b + 6\,a^2\,b^2 - 4\,a\,b^3 + b^4}{a^2 - 2\,a\,b + b^2} = \dfrac{(a-b)^4}{(a-b)^2}$

$$= (a-b)^2 = a^2 - 2\,a\,b + b^2.$$

(5) $x^{\frac{5}{3}} + x^{\frac{4}{3}}\,y^{\frac{1}{3}} + x\,y^{\frac{2}{3}} + x^{\frac{2}{3}}\,y + x^{\frac{1}{3}}\,y^{\frac{4}{3}} + y^{\frac{5}{3}}$.

(6) $(1 + x - x^2)^2 - (1 - x + x^2)^2$

$$= \{1 + (x - x^2)\}^2 - \{1 - (x - x^2)\}^2$$

$$= \{1 + x\,(1 - x)\}^2 - \{1 - x\,(1 - x)\}^2$$

$$= 1 + 2\,x\,(1 - x) + x^2\,(1 - x)^2 - 1 + 2\,x\,(1 - x) - x^2\,(1 - x)^2$$

$$= 4\,x\,(1 - x).$$

8. (1) $x = 4$. (2) $x = 60$. (3) $x = 21$. (4) $x = 42$, $y = 24$.

9. (1) Art. 112.

Then a, $a + d$, $a + 2\,d$, $a + 3\,d$, $a + 4\,d$, $a + 5\,d$, will be six terms of the series.

(2) 62. (3) 20300. (4) P. 59. Apply Ex. 6.

The series is 17, 15, 13, 11, 9, 7, 5, 3.

(5) $S = \frac{a\,r^n - a}{r - 1}$, for a geometrical series,

$$= \frac{\frac{1}{2} \times \frac{1}{2}^5 - \frac{1}{2}}{\frac{1}{2} - 1} = \frac{\frac{1}{64} - \frac{1}{2}}{-\frac{1}{2}} = \frac{-\frac{31}{64}}{-\frac{1}{2}} = \frac{31}{32}.$$

(6) When n is infinite, $S = \frac{a}{1 - r} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1.$

$\therefore$ the sum approaches to unity, when the number of terms is increased without limit.

10. (1) 7 : 63 : : ·015 : ·135.

(2) Since $a\,x - b\,y = c\,x + d\,y$, $(a - c)\,x = (b + d)\,y$,

$$\therefore x : y : : b + d : a - c.$$

(3) [1] Since $a : b : : c : d$, $\frac{a}{b} = \frac{c}{d}$, and $\frac{a^2}{b^2} = \frac{c^2}{d^2}$,

$$\therefore a^2 : b^2 : : c^2 : d^2.$$

[2] Again *alternando*, $\frac{a^2}{c^2} = \frac{b^2}{d^2}$; and since $\frac{a^2}{b^2} = \frac{c^2}{d^2}$, add 1 to each side.

$$\therefore \frac{a^2}{b^2} + 1 = \frac{c^2}{d^2} + 1, \frac{a^2 + b^2}{b^2} = \frac{c^2 + d^2}{d^2}, \frac{a^2 + b^2}{c^2 + d^2} = \frac{b}{d^2} = \frac{a^2}{c^2},$$

$$\therefore a^2 + b^2 : c^2 + d^2 : : a^2 : c^2.$$

(4) Since $\frac{a}{b} = \frac{\alpha}{\beta}$, $\frac{b}{c} = \frac{\beta}{\gamma}$, $\frac{c}{d} = \frac{\gamma}{\delta}$,

$$\therefore \frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} = \frac{\alpha}{\beta} \times \frac{\beta}{\gamma} \times \frac{\gamma}{\delta},$$

Whence $\frac{a}{d} = \frac{\alpha}{\delta}$; and $a : d : : \alpha : \delta$.

1855. *July 3rd.*

1. P. 47, Ex. 9.

2. (1) Art. 39.

(2) $2 \times 2 \times 3 \times 3 \times 5 \times 11 \times 13 = 25740$
$= 2^2 \times 3^2 \times 5 \times 11 \times 13$. *Ans.*

(3) Art. 38.

3. (1) £616 4*s.* 9·9*d.* (2) £11 19*s.* 7·7925*d.*

(3) To find the rate per cent. we have
(1*s.* 2*d.*) 100 = 1400*d.* = £5 16*s.* 8*d.* = £5$\frac{5}{6}$.

4. (1) The least common multiple of two or more numbers, is the least number which will contain each of the given numbers an exact number of times without a remainder.

(2) $592 = 16 \times 37,$
$1369 = 37 \times 37.$

Since 16 and 37 are prime to each other, it is clear that 37 is the greatest common measure of 592 and 1369; therefore their least common multiple must contain 16 and 37 × 37, as factors, Now every multiple of 592 must contain 16 and 37 as factors; and every multiple of 1369 must contain 37 × 37 as a factor; therefore every number which is a multiple of 592 and 1369 must contain 16 and 37 × 37, as factors; and the least number which so contains them is 16 × 37 × 37, or 21904.

Now 21904 = (16 × 37) (37 × 37) divided by 37
= 592 × 1369 divided by 37
= 592 × 1369 divided by the greatest common measure of 592 and 1369.

(3) $\frac{308}{420}, \frac{357}{420}, \frac{380}{420}, \frac{345}{420}$. (4) $\frac{1}{16}$. (5) $\frac{3}{112}$.

5. (1) Arts. 83, 84. (2) Art. 89. (3) [1] 34·75. [2] ·7777.
(4) [1] *6·775. [2] ·015974. [3] 35·75. [4] 130000. [5] 73·45.

For an explanation of the process of borrowing and carrying, see Art. 11, Note.

6. (1) and (2) P. 45, Ex. 4. (3) $\frac{79}{165}$.

(4) From $\cdot 3333333\dot{3}$
Take $\cdot 121212\dot{1}\dot{2}$

Diff. = $\cdot 212121\dot{2}\dot{1}$

7. (1) and (2) Art. 7. (3) $8x + 3y$.
(4) $21x^4 - 37x^3y + 18x^2y^2 - 26xy^3 + 24y^4$.
(5) $3a^2 + 4ab - b^2$. (6) $\frac{4ab}{(a+b)^2}$.

8. (1) $x = 12$. (2) $x = 5$. (3) $x = 7, y = 5$. P. 71, Ex. 4.
(4) P. 71, Ex. 5. $x = 12, y = 15$.

9. Here 5 : ·05 : : ·05 : ·0005.
(2) $77x + 55y = 217x - 155y$, $210y = 140x$, $3y = 2x$.
$\therefore x : y :: 3 : 2$.

(3) [1] First $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, and if $\frac{a}{b} = r$, then $\frac{e}{f} = r$;

and $a = br, e = fr \therefore \frac{a}{e} = \frac{br}{fr} = \frac{b}{f}$.

[2] Solutions *M.*, 1849, Quest. 10.

10. (1) − 3, 1, 5, 9, 13, 17. (2) 279. (3) P. 59, Ex. 4.
(4), (5), &c., p. 63, Ex. 13.

1856. *July 8th.*

1. (1) In 2 days. (2) Art. 101.
2. (1) £465 7*s.* $0\frac{3}{10}$*d.* (2) $3\frac{1}{2}$ per cent.
3. (1), (2), (3) Art. 40. (4) 117. (5) The numbers are prime to each other when unity is their greatest common measure.

4. (1) Art. 81. (2) $1\frac{7}{8}$. (3) Art. 78.
(4) $\frac{11}{12}$ must be divided by $\frac{11}{36}$ to give a quotient 3.
(5) The earth's diameter $= \frac{69\frac{1}{22} \times 360 \times 7}{22} = 7908{\cdot}84297$ miles.

5. (1) ·5, ·17, ·0006. (2) Art. 83. (3) [1] 0·6046.
[2] 33·738562. [3] ·275. [4] 7·205.

6. (1) Art. 87. (2) Art. 88. (3) Art. 88, Note. (4) $\frac{226}{495}$.

7. (1) $5^4 = 625$. (2) $2a - c$.
(3) $a^6 - 5a^5b + 11a^4b^2 - 17a^3b^3 + 15a^2b^4 - 14ab^5 + 3b^6$.
(4) $4x^2 - 7x + 5$.
(5) $$1 + \frac{b^2 - c^2 - a^2}{2bc} = \frac{2bc + b^2 + c^2 - a^2}{2bc}$$
$$= \frac{(b+c)^2 - a^2}{2bc} = \frac{(b+c+a)(b+c-a)}{2bc}.$$

8. (1) $x = 24$. (2) $x = 21$. (3) $x = 7, y = 2$.
(4) $x = 144, y = 216$.

9. (1) Art. 101, $ad = bc$. (2) [1] Art. 146.
[2] Arts. 145 and 147. (3) P. 82, Ex. 6.
(4) Let $a = 3$; $b = 6$.
Then $a : b :: b : c$; or, $3 : 6 :: 6 : 12$;
and $a : c :: a^2 : b^2$; or, $3 : 12 :: 9 : 36$.

10. (1) $\frac{6 + 24}{2} = 15$. $\sqrt{6 \times 24} = 12$. (2) 438. (3) $\frac{7}{5}$. (4) 484.

1857. *July 7th.*

1. P. 47, Ex. 10. *Ans.* 50 square chains.

2. (1) £334 10*s.* $1\frac{3}{5}$*d.* (2) $2\frac{8}{21}$ per cent.

3. (1) 410 and 615.

Also 205 is the *g. c. m.* of any two numbers which are multiples of itself by numbers prime to each other.

(2) 113. (3) Art. 41. (4) $\frac{50}{60}, \frac{35}{60}, \frac{36}{60}, \frac{51}{60}, \frac{41}{60}$.
(5) $\frac{300}{360}, \frac{210}{360}, \frac{216}{360}, \frac{306}{360}, \frac{246}{360}$.

4. (1) Art. 76. (2) Let $x =$ the number.
Then $(\frac{1}{3} + \frac{1}{4} + \frac{1}{6})\,x = 45$; $\frac{3}{4}x = 45$; $x = 60$.
(3) $\frac{143}{140}$.

5. (1) Art. 83. (2) $\cdot45 = \frac{45}{100}$ of the unit. (3) $\cdot00007 = \frac{7}{100000}$.
(4) Arts. 90 and 91. (5) $33{\cdot}175 = \frac{33175}{1000}$. $\cdot0008 = \frac{8}{10000}$.
And $\frac{33175}{1000} \times \frac{8}{10000} = \frac{265400}{10000000} = \frac{2654}{100000} = \cdot02654$.
(6) [1] ·03965. [2] $\sqrt{\frac{14{\cdot}4}{16{\cdot}9}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$. [3] 57·04.
(7) Art. 98.

6. (1) P. 44, Ex. 2.

(2) 5·5309. The true value of the remainder is ·0083.

(3) $\frac{832}{990}$. Art. 89. (4) $\cdot 000\dot{9} = \frac{9}{9000} = \frac{1}{1000} = \cdot 001$.

7. (1) Art. 15. (2) Arts. 16 and 17. (3) $\frac{a^2 + 6\,a\,b - 3\,b^2}{4}$.

(4) $(a - b)^5 \times (a - b)^2 = (a - b)^7$

$= a^7 - 7\,a^6\,b + 21\,a^5\,b^2 - 35\,a^4\,b^3 + 35\,a^3\,b^4 - 21\,a^2\,b^5 + 7\,a\,b^6 - b^7$.

(5) $x^4\,y^4 + x^3\,y^3 + x^2\,y^2 + x\,y + 1$.

(6) To reduce $(1 + 2\,x + 3\,x^2)^2 - (1 - 2\,x + 3\,x^2)^2$,
let $(1 + 3\,x^2) = a$; $2\,x = b$;
then the expression becomes

$(a + b)^2 - (a - b)^2 = 4\,a\,b = 4\,(1 + 3\,x^2) \times 2\,x = 8\,x + 24\,x^3$.

8. (1) $x = 3$. (2) $x = 42$. (3) $x = \frac{2}{3}$. (4) P. 71, Ex. 8.

(5) P. 67, Ex. 9.

9. (1) From Art. 113, $d = \frac{l - a}{n - 1} = \frac{65 - 20}{3} = 15$;

$\therefore$ the terms are 20, 35, 50, 65.

(2) $-21, -13, -5, 3, 11, 19, 27, 35, 43$. (3) 567.

(4) 426. (5) $8\frac{2}{3}$. (6) Art. 120; 2 the limit required.

(7) Multiplying extremes and means,

$$21\,a\,c + 35\,a\,d + 3\,b\,c + 5\,b\,d = 21\,a\,c + 3\,a\,d + 35\,b\,c + 5\,b\,d.$$

$$\therefore 32\,a\,d = 32\,b\,c.$$

$$\therefore \frac{a}{b} = \frac{c}{d},$$

or $a : b :: c : d$.

1858. *July 6th.*

1. (1) $\frac{3}{4}$. (2) 62·208. (3) Art. 91.

2. 951. Art. 96.

3. (1) £82 3*s.* 3*d.* (2) $2\frac{1}{2}$. P. 87, Ex. 4.

4. P. 76, Ex. 8.

5. (1) $\frac{2\,(a^2 + b^2)}{a^2 - b^2}$. (2) $\frac{-4\,a\,b}{a^2 - b^2}$.

(3) Art. 14. The *origin* of negative quantities arose from the necessity of *representing* results unattainable by simple arithmetic, as the subtraction of 30 from 10, which is expressed by -20.

6. (α) $x = 14$. (β) $x = 11$. (γ) P. 72, Ex. 9.

7. (1) Art. 100, see *Euclid*, Book V., definition 3.

(2) By considering their equality or inequality (see Art. 101 and *Euclid*, Book V., definitions 5, 6, 7).

(3) $\frac{1}{2}$ is a ratio of *less* inequality than $\frac{1}{4}$; add 2 to num. and den. of both, then $\frac{3}{4}$ becomes a ratio of *greater* inequality than $\frac{1}{3}$.

(4) Art. 145.

8. (1) Art. 114 (III.). Given $a = \frac{1}{2}$, $d = -\frac{5}{4}$, $n = 24$.

$\therefore S = \frac{(1 + 23\,(-\frac{5}{4}))\,24}{2} = -\frac{111}{4} \times 12 = -333$. *Ans.*

(2) Art. 114 (II.). Given $a = 1$, $l = 50$, and $S = 204$.

$204 = \frac{51\,n}{2}$. $\therefore n = 8$, and by I., $50 = 1 + 7\,d$.

$\therefore d = 7$. *Ans.*

9. The ratio of the series is $\left(\frac{c}{a}\right)^{\frac{1}{n+1}}$; and the means are $a\left(\frac{c}{a}\right)^{\frac{1}{n+1}}$, $a\left(\frac{c}{a}\right)^{\frac{2}{n+1}}$, &c. . . . $a\left(\frac{c}{a}\right)^{\frac{n-1}{n+1}}$, $a\left(\frac{c}{a}\right)^{\frac{n}{n+1}}$. The product of these is $a^n\left(\frac{c}{a}\right)^{\frac{1+2+3+\&c.\ldots n-1+n}{n+1}}$, or (Art. 114, III.), $a^n\left(\frac{c}{a}\right)^{\frac{\overline{n+1}\,.\,n}{2\,.\,\overline{n+1}}}$, which $= a^n\left(\frac{c}{a}\right)^{\frac{n}{2}} = (a\,c)^{\frac{n}{2}}$.

1859. *July 5th.*

1. (1) As £650½ : £2450⅓ : : £24 7*s.* 10½*d.* : £91 17*s.* 9*d.*
   (2) As £650 10*s.* : £24 7*s.* 10½*d.* : : £1 : 9*d.*
2. (1) £697 2*s.* 6*d.* Art. 155, I. (2) 7½ years. Art. 156, III.
3. (1) Arts. 75, 6. (2) $\frac{1}{9}$. (3) Ex. 11, p. 48.
4. (1) Art. 83. (2) $86{\cdot}5 \times {\cdot}00164 = {\cdot}14186$. (3) $\frac{11{\cdot}3}{28{\cdot}25} = {\cdot}4$.
5. (1) Art. 95. (2) Art. 99, ¶ 3. (3) 35·01 and $\sqrt{18{\cdot}\dot{7}} = \dot{4}{\cdot}3 = 4\frac{1}{3}$.
6. $2\frac{1}{6}$ and 76. Yes.
   (1) $5\,b^2 + c^2$. (2) $a^5 + a^4\,x - 2\,a^3\,x^2 - 2\,a^2\,x^3 + a\,x^4 + x^5$.
   (3) $x^2 - x - 2$. (4) $4\,x\,(1 + x^2)$.
7. (1) $x = 68$. (2) $x = 14$. (3) $x = 8$; $y = 9$.
8. Ex. 1, p. 75.
9. (1) 1·5 : ·09 : : ·45 : ·027.
   (2) $\because \frac{a}{b} = \frac{c}{d}$, and $\frac{e}{f} = \frac{g}{h}$.

$$\therefore \frac{a}{b} \times \frac{e}{f} = \frac{c}{d} \times \frac{g}{h}, \quad \therefore \quad a\,e : b\,f : : c\,g : d\,h.$$

(3) $x^2 - y^2 : x^2 + y^2 : : 5 : 13$.

$$\therefore 13\,x^2 - 13\,y^2 = 5\,x^2 + 5\,y^2,$$
$$8\,x^2 = 18\,y^2,$$
$$\frac{x^2}{y^2} = \frac{9}{4}, \therefore \frac{x}{y} = \frac{3}{2},$$
$$\text{or } x : y : : 3 : 2.$$

10. (1) $a - 3\,b$, $a - 2\,b$, $a - b$.
    (2) The 7th term $= \frac{4}{5} - 6 \times \frac{1}{5}$ (Art. 113, I.) $= -\frac{2}{5}$.

(Art. 114, IV.) $S = \frac{(-\frac{4}{5} - (-\frac{6}{5}))\,7}{2} = \frac{(\frac{6}{5} - \frac{4}{5})\,7}{2} = \frac{7}{5} = 1\frac{2}{5}$.

(3) $4 + 5d = 29 \quad \therefore d = 5.$

$\therefore$ 9, 14, 19, 24, terms required.

(4) Arts. 120, 121, and see Ex. 4, p. 62.

(5) $\sqrt{\left(\frac{a+b}{2} \times \frac{2ab}{a+b}\right)} = \sqrt{ab}.$

1860. *January 10th.*

1. 48248.

2. (1) Art. 37, ¶ 4. (2) 6435, 4356, 3465, 5346, 6534, 4653, 3564, 5643, are all divisible by 11, and no other numbers composed of the same digits are so divisible.

3. $4\frac{1}{2}$ per cent.

4. (1) Arts. 80, 81. (2) Given fraction =

$$\frac{\frac{2}{3} + \frac{4}{9} - \frac{8}{21}}{1 + \frac{2}{7} - \frac{5}{9}} = \frac{\frac{42 + 28 - 24}{63}}{\frac{63 + 18 - 35}{63}} = \frac{46}{63} \div \frac{46}{63} = 1.$$

5. (1) ·40708. (2) ·625. (3) $\cdot 1\dot{8}5714\dot{2}$. (4) Yes. Art. 88.

6. (1) 2296. (2) 23·002.

7. (1) 1. (2) Because $x - 4 = -(4 - x)$, and $(+a)^2 = (-a)^2$. [1] $2a + 5\frac{2}{3}b - \frac{3c}{4}$.

[2] $x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6$.

[3] $3x - 5y$.

8. (1) $x = 16$. (2) $x = 20, y = 12$.

9. (1) P. 77, Ex. 11. (2) $\frac{a^2 + b^2}{2}$.

(3) $\because \frac{a}{b} = \frac{c}{d} \quad \therefore \frac{a^2}{b^2} = \frac{c^2}{d^2},$

$$\frac{a^2 + c^2}{b^2 + d^2} = \frac{a^2}{b^2} \quad \therefore \frac{\sqrt{a^2 + c^2}}{\sqrt{b^2 + d^2}} = \frac{a}{b};\text{—i.e.,}$$

$$(a^2 + c^2)^{\frac{1}{2}} : (b^2 + d^2)^{\frac{1}{2}} :: a : b.$$

1860. *July 3rd.*

1. (1) P. 92, Ex. 9. (2) $1\frac{1}{4}$.

2. $\frac{3}{8}$ of 13*s.* 4*d.* $= \frac{13\frac{1}{3} \times 3}{8} =$ 5*s.*, which brought to the fraction of a guinea $= \frac{5}{21}$; $\frac{2}{9}$ of 7*s.* 6*d.* $= \frac{7\frac{1}{2} \times 2}{9} =$ 1*s.* 8*d.*; $\cdot\dot{1}$ of 9*s.* 9*d.* $= \frac{1}{9} \times 9\frac{3}{4} = \frac{39}{36} =$ 1*s.* 1*d*

The numerator $\frac{1}{1-\frac{2}{3}} - \frac{1}{3} + \frac{3}{2} = \frac{3}{1} - \frac{1}{3} + \frac{3}{2} = \frac{18-2+9}{6} = \frac{25}{6}$.

The denominator $3\left(1 + \frac{4}{7}\right) - 4 = 3 + \frac{12}{7} - 4 = \frac{21+12-28}{7} = \frac{5}{7}$,

$$\text{and } \frac{25}{6} \div \frac{5}{7} = \frac{25}{6} \times \frac{7}{5} = \frac{35}{6} = 5\tfrac{5}{6}.$$

3. (1) Art. 89. (2) $2\frac{413}{990}$. (3) $133\cdot\dot{3}$. (4) $13333\cdot\dot{3}$.
   (5) $\cdot 25^{\cdot 5} = \cdot 25^{\frac{1}{2}}$ (Art. 82) $= \sqrt{\cdot 25} = \cdot 5$.
4. In the first case $\frac{2}{3}$ of every ℔ will be at 5*s.* and $\frac{1}{3}$ at 4*s.*
   $\therefore\ 5 \times \frac{2}{3} + 4 \times \frac{1}{3}$, or $4\frac{2}{3}$ = 4*s.* 8*d.*
   Second case $5 \times \frac{1}{3} + 4 \times \frac{2}{3}$, or $4\frac{1}{3}$ = 4*s.* 4*d.*
5. (1) $2\frac{1}{2}x^2$. (2) 1. (3) $(1+x)(1-3x)$. (4) $3\frac{5}{8}$.
6. (1) $x = 6$. (2) $x = 7;\ y = 3$.
7. (1) 2·01. (2) $x^2 + 2x - 1$.
   (3) $x^2 + 2ax + b^2 = x^2 + 2cx + c^2$.
   $$\therefore\ 2(a-c)x = c^2 - b^2.$$
   $$x = \frac{c^2 - b^2}{2(a-c)}.$$
   (4) If $a = b = c$, the equation becomes $x = \frac{0}{0}$, *i.e.*, every possible value of x satisfies the conditions of the problem.
   (5) For $a^2 - 1 = \overline{a+1} \cdot \overline{a-1}$.
8. (1) Art. 103. (2) $1 + x : 2 + x :: 8 - x : 10 - x$.
   $$\therefore\ 10 + 9x - x^2 = 16 + 6x - x^2,$$
   $$3x = 6 \text{ and } x = 2.$$
   And the proportion is $3 : 4 :: 6 : 8$.
   (3) $\because\ a : b :: c : d\ \therefore\ a : c :: b : d$.
   $$\therefore \frac{a}{c} = \frac{b}{d}\ \therefore\ \frac{a+b}{c+d} = \frac{b}{d},$$
   $$\frac{a^3}{c^3} = \frac{b^3}{d^3} = \left(\frac{a+b}{c+d}\right)^3 = \frac{a^3+b^3}{c^3+d^3}.$$
   $$\therefore \frac{a^3+b^3}{c^3+d^3} \cdot \frac{b}{d} = \left(\frac{a+b}{c+d}\right)^3 \cdot \frac{b}{d} = \left(\frac{a+b}{c+d}\right)^4.$$
   (4) $\frac{6 \times 12 \times 7 \times 6}{14 \times 3 \times 2} = 36$.
9. (1) Arts. 112 and 116. (2) Increased. (3) $40\frac{5}{8}$. (4) $1\frac{1}{3}$.

1861. *January 16th.*

1. (1) 27720. See Art. 48.
   (2) The reason is, that annexing 4 ciphers multiplies by 10000, and 10000 ÷ 16 = 625.

(3) $70354 \times 625 = 703540000 \div 16 = 43971250$.

2. (1) $\frac{25}{57} - \frac{377}{1033}$ or $\frac{4336}{58881}$.

(2) £71 2*s.* = £71·1, and $17{\cdot}975 \times 71{\cdot}1 = 1278{\cdot}0225$ = £1278 0*s.* 5$\frac{2}{5}$*d.*; also £51120 18*s.* ÷ 40 = £1278 0*s.* 5$\frac{2}{5}$*d.*

3. (1) 666. (2) ·00067.

(3) $\sqrt{\cdot\dot{4}} = \cdot\dot{6}'$ as it evidently ought to be; since $\cdot\dot{4} = \frac{4}{9}$,

$$\text{and } \sqrt{\frac{4}{9}} = \frac{2}{3} = \cdot\dot{6}.$$

(4) 80. (5) $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1{\cdot}414213\ldots}{2} = \cdot707106\ldots$

4. (1) Arts. 155, 159. (2) £792 15*s.*

(3) The $3\frac{1}{2}$ per cents., since they yield 3·66013 . . . per cent. on the money invested, and the others 3·65853 . . .

5. (1) Sum $= 9\,(a + b + c)$ difference $= a + b + c$; product of these is $9\,(a^2 + 2\,a\,b + 2\,a\,c + b^2 + 2\,b\,c + c^2)$.

(2) $\frac{3\,x^2}{1 + 2\,x + 2\,x^2 + x^3}$.

(3) $(x^2 + a\,x + a^2)\,(x^2 - a\,x + a^2) = x^4 + a^2\,x^2 + a^4$.

6. (1) $x = 110$. (2) $x = a \pm c$.

(3) $x = \frac{2\,a\,b}{b - a}$, $y = \frac{2\,a\,b}{b + a}$. (4) $x = \frac{a^3 - c^3}{3\,c^2 - a^2}$.

(5) Let the two numbers be a and b. The difference between the cubes of the sum and difference of these is $6\,a^2\,b + 2\,b^3$, which is divisible by $3\,a^2 + b^2$, giving as the quotient $2\,b$.

7. Let the person walk x times as fast as the waggon moves; then the distance between the person and the tail of the waggon which he meets will, at the time at which he first sees it, be $\left(20 + \frac{20}{x}\right)$ yards. Similarly the same distance will in the other case be $\left(60 - \frac{60}{x}\right)$ yards.

$$\therefore 20 + \frac{20}{x} = 60 - \frac{60}{x}, \text{ or, } \frac{80}{x} = 40.$$

$$\therefore x = 2.$$

And consequently the distance of the person from the farthest part of the waggon in the first case, and from the head of the foremost horse in the second, will be 30 yards; therefore, the distance he can see in the fog is 30 yards, *minus* the length of the waggon and horses.

8. (1) Art. 145. (2) $\because a : b :: c : d. \quad \therefore \frac{a}{b} = \frac{c}{d}.$

$$\therefore \frac{a\,c}{b\,d} = \frac{a^2}{b^2} = \frac{c^2}{d^2}.$$

$$\text{Also } \frac{a+c}{b+d} = \frac{a}{b}. \quad \therefore \frac{a\,c}{b\,d} \cdot \frac{a+c}{b+d} = \frac{a^3}{b^3} = \frac{c^3}{d^3} = \frac{a^3+c^3}{b^3+d^3},$$

$$\therefore a\,c\,(a+c) : b\,d\,(b+d) :: a^3+c^3 : b^3+d^3,$$

$$\text{or } a^2 c + a\,c^2 : b^2 d + b\,d^2 :: a^3+c^3 : b^3+d^3.$$

(3) If $a+b+c+d : a+b-c-d :: b+d : b-d$, then $(b-d)(a+b)+(b-d)(c+d) = (b+d)(a+b)-(b+d)(c+d)$.

$$\therefore 2\,b\,(c+d) = 2\,d\,(a+b).$$

$$\therefore a\,d + b\,d = b\,c + b\,d, \text{ or } a\,d = b\,c.$$

$$\therefore a : b :: c : d.$$

9. (1) See Art. 114, III. (2) $\frac{(29-20)\,50}{2} = 225.$

(3) $\frac{1}{1+\frac{1}{3}} = \frac{3}{4}.$ (4) $\frac{1}{9} \div 1 - \frac{1}{81} = \frac{1}{9} \div \frac{80}{81} = \frac{9}{80}.$

1861. *July 3rd.*

1. (1) $65\frac{5}{27}$ yards. (2) $10\frac{10}{11}d.$ per ℔.
2. (1) Art. 83. (2) Arts. 92, 93. (3) $\cdot\dot{5}3846\dot{1}$. (4) 18000.
3. 3000 days.
4. (1) $\frac{1}{x-1}.$ (2) $\frac{x-1}{x-2}.$

(3) $-(a_0 + a_1 + a_2 + a_3 + \ldots)$ or the negative sum of the co-efficients.

5. (1) $x = 4\frac{2}{3}.$ (2) $x = 4, y = 2.$
6. Arts. 112, 116. (1) $(a+l) \div 2.$ (2) $\sqrt{a\,l}.$

(3) $\frac{(2\,a + \overline{n-1}\,.\,a\,r)\,n}{2}.$ (4) $\frac{a\,(\overline{1+r}^{\,n}-1)}{r}.$

(5) $\frac{(2+\frac{n-1}{2})\,n}{2}$, or $\frac{(3+n)\,n}{4}.$

7. (1) P. 85,* Ex. 8. (2) Art. 145.

(3) $\frac{a}{b} = \frac{c}{d} \quad \therefore \frac{a^2}{b^2} = \frac{a\,c}{b\,d}.$

$$\therefore \frac{a}{b} = \frac{\sqrt{a\,c}}{\sqrt{b\,d}}. \quad \therefore \frac{a+b}{b} = \frac{\sqrt{a\,c}+\sqrt{b\,d}}{\sqrt{b\,d}}.$$

$$\text{Also} \qquad \frac{a-b}{b} = \frac{\sqrt{a\,c}-\sqrt{b\,d}}{\sqrt{b\,d}}.$$

$$\therefore \frac{a+b}{a-b} = \frac{\sqrt{a\,c}+\sqrt{b\,d}}{\sqrt{a\,c}-\sqrt{b\,d}}.$$

$$\therefore a+b : a-b :: \sqrt{a\,c}+\sqrt{b\,d} : \sqrt{a\,c}-\sqrt{b\,d}.$$

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* See remark (taken from *Univ. Cal.* for 1872) appended to Solution to Question 9, June 1871.

8. (1) $x^2 - x + 1$.

(2) The fourth root is the square root of $x + 2 + \frac{1}{x} = \sqrt{x} + \frac{1}{\sqrt{x}}$.

9. Since A travels $\frac{4}{3}$ of the distance from P to Q, while B travels only $\frac{2}{3}$, $\therefore$ A travels twice as fast as B; say A travels $2\,x$ per hour, and B, x . . In a hours after meeting B, A will have travelled $\frac{4}{3}$ of the distance between P and Q, and $2\,a\,x$ miles more, say $\frac{4\,d}{3} + 2\,a\,x$. Hence C travels at the rate of $\frac{(2\,d - 6\,a\,x)\,x}{2\,d + 3\,a\,x}$ miles an hour. In b hours after meeting A, B will have travelled $\frac{2\,d}{3} + b\,x$ miles. Hence C travels $\frac{(2\,a - b)\,x}{b - a}$ miles per hour. Therefore,

$$\frac{(2\,d - 6\,a\,x)\,x}{2\,d + 3\,a\,x} = \frac{(2\,a - b)\,x}{b - a}, \text{ and } x = \frac{4\,b\,d - 6\,a\,d}{3\,a\,b}.$$

Therefore, finally, the time when A and B first meet is $\frac{a\,b}{2\,b - 3\,a}$ hours after starting from P.

1862. *January* 15*th*.

1. £723; rate of interest is 3·3195 per cent.

2. *Ans.* £38 2*s.*, £25 8*s.*, and £63 10*s.* respectively, exclusive of interest; or, £60, £40, and £100, including interest.

3. (1) and (2) Arts. 49, 65.

(3) $\frac{240}{193} = 1\frac{41689}{171191}$, $\frac{1103}{887} = 1\frac{41688}{171191}$. $\therefore$ the former is the greater.

(4) $\frac{13}{30}$. (5) $\frac{23}{5}$, or $4\frac{3}{5}$.

4. (1) and (2) Arts. 83, 85. (3) ·40305. (4) ·00390625.

(5) 18*s.* $10\frac{398}{625}$*d.* (6) ·028125, or $\frac{9}{320}$.

5. (1) and (2) 0 in both cases. (3) $-2\,a + 3\,b$.

(4) $-4\,x^2\,z^2$. (5) $\frac{x^4 - 4\,x^2 + 4}{2\,x^2 - 4\,x + 4}$.

(6) $x^5 - 5\,x^4\,a + 10\,x^3\,a^2 - 10\,x^2\,a^3 + 5\,x\,a^4 - a^5$.

6. (1) $x = 4$. (2) $x = \frac{1}{2}\;y = \frac{1}{3}$.

7. Arts. 101, 146. (2) Art. 145.

(3) $\frac{a^3 - a^2\,b - a\,b^2 + b^3}{a^2 + b^2}$.

8. (1) Art. 117. (2) 2, and p. 63, Ex. 11. (3) Art. 120.

(4) $\frac{2\,n^2 + 3\,n}{2}$.

(5) Here are two series; $1 + 2 + 3 + \ldots$ to n terms, and $-\frac{3}{2} - \frac{5}{2} - \ldots$ to n terms: s_n of the first $= \frac{n^2 + n}{2}$; s_n of the second $= -\frac{n^2 + 2\,n}{2}$, sum of both $= -\frac{n}{2}$.

9. See 1841, October 4, Ques. 10; or, since 3 of the policeman's steps = 5 of the boy's, 1 of the policeman's = $1\frac{2}{3}$ of the boy's; and since he takes 2 while the boy takes 3, he goes $3\frac{1}{3}$ boy's steps to the boy's 3. Of course the boy will be overtaken.

1862. *July 9th.*

1. (1) See Arts. 49, 70, 77.

(2) Sum $= 1\frac{3}{8}$. Divisor $= \frac{143}{315}$. Quotient $= 3\frac{3}{104}$.

2. (1) £1 1*s.* $5\frac{1}{2}$*d.*

(2) 3 miles 2 fur. 100 yds. $= 26\frac{5}{11}$ fur.; 12 leagues 2 miles 2 fur. 20 yds. $= 306\frac{1}{11}$; $26\frac{5}{11} \div 306\frac{1}{11} = \frac{291}{3367}$.

3. (1) 5*s.* (2) £1 17*s.* 6*d.*

4. (1) £1050. (2) $3\frac{1}{2}$ per cent.

5. (1) Arts. 85 . . 8. (2) ·00000146832. (3) ·0456.

(4) £10 10*s.* (5) £66.

6. (1) Sum $= 2\,x + 4\,y - 2\,z$; product $= 2\,x^2 + 2\,x\,y - 4\,y^2 - 4\,x\,z - 2\,y\,z + 2\,z^2$. (2) $\frac{8\,(y + 2)}{8\,y - 1}$.

(3) Product and quotient alike $= x^7 + x^6\,y + x^5\,y^2 + x^4\,y^3 + x^3\,y^4 + x^2\,y^5 + x\,y^6 + y^7$.

(4) $\frac{1}{x^3 - 6\,x^2 + 11\,x - 6}$.

7. (1) Art. 145. (2) $\frac{a}{b} = \frac{c}{d}$. $\therefore \frac{a^2}{b^2} = \frac{c^2}{d^2} = \frac{a^2 + c^2}{b^2 + d^2}$.

$$\therefore \left(\frac{a^2 + c^2}{b^2 + d^2}\right)^2 = \frac{a^4}{b^4} = \frac{c^4}{d^4} = \frac{a^4 + c^4}{b^4 + d^4}.$$

$$\therefore \frac{a^2 + c^2}{b^2 + d^2} = \frac{\sqrt{a^4 + c^4}}{\sqrt{b^4 + d^4}}.$$

$$\therefore a^2 + c^2 : b^2 + d^2 :: \sqrt{a^4 + c^4} : \sqrt{b^4 + d^4}.$$

(3) Let $\frac{a}{b}$ become $\frac{a + m}{b + m}$: then $\frac{a}{b}$ is greater or less than

$\frac{a + m}{b + m}$ according as $a > b$, or $a < b$. For $\frac{a}{b} - \frac{a + m}{b + m}$, or $\frac{(a - b)\,m}{b\,(b + m)}$ is evidently positive or negative, according as a is greater or less than b. If $a = b$, this difference vanishes; or, $\frac{a}{b} = \frac{a + m}{b + m}$.

8. (1) See Arts. 112, 113, and 114, III. (2) $67\frac{1}{2}$.
(3) $913\frac{1}{2}$. (4) First term $= 4\frac{1}{2}$. $\therefore s = \frac{(2\,n + 7)\,n}{2}$.

9. (1) $x = 12$. (2) $x = 5$, $y = 11$.
(3) $7\,x^3 + 9\,a\,x^2 + 3\,a^2\,x = 4\,a\,(7\,x^2 + 9\,a\,x + 3\,a^2)$ $\therefore x = 4\,a$.

10. Let $x =$ number of £ taken; $\therefore x + 295 =$ number of persons admitted by season tickets; also, $20\,x + 7 =$ number of persons that entered by payment. $\therefore 21\,x + 302 = 58682$, and $x = 2780$. $\therefore$ 3075 entered by season tickets, and 55607 by payment.

1863. *January* 14*th*.

1. He loses 80 per cent.

2. (1) $99\frac{279}{333} \times 1\frac{2}{3} = \frac{3694}{37} \times \frac{5}{3} = 166\frac{44}{111}$. Of course 166 is the nearest integer. (2) Both $= 1d$. (3) 1.

3. (1) ·078. (2) 800. (3) 10*s*. $7\frac{1}{2}d$.

4. (1) £1299·2523364486. (2) In $20\frac{5}{32}$ years, or $3\frac{11}{32}$ years sooner.

5. (1) $\frac{a + b + c}{2}$, or s. (2) $\frac{3\,x + 2}{x - 1}$.
(3) $x^8 - x^7 y + x^6 y^2 - x^5 y^3 + x^4 y^4 - x^3 y^5 + x^2 y^6 - x\,y^7 + y^8$.
(4) $a^4 - b^4$.

6. (1) $x = 2$. (2) $x = -3\frac{4}{5}$. (3) $x = \frac{a'\,b - a\,b'}{b - b'}, y = \frac{a'\,b - a\,b'}{a' - a}$.
(4) At $31\frac{3}{7}$ minutes past 4.

7. (1) and (2) Arts. 101, 102. (3) Equating any two of the given fractions, and removing the denominators in the usual way, (thus, taking the first and the third, $a\,d + b\,c - b\,f - f^2 + f\,d = - a\,d - b\,c + b\,f - c^2 + c\,d$, or $2\,a\,d + 2\,b\,c - 2\,b\,f = f^2 - c^2 + c\,d - f\,d$,) there will result an equation of which one obvious solution is $b = 0$, $d = 0$, $f = -c$. Substitution gives

$$\frac{a - c}{a - c} = \frac{a - c}{a - c} = \frac{2\,c}{2\,c}.$$

8. (1) P. 58, Art. 114, III.

$$s = \frac{(2\,a + (n-1)\,d)\,n}{2} \quad \therefore \frac{s}{n} = a + \frac{(n-1)\,d}{2};$$

$$\text{and } a = \frac{s}{n} - \frac{(n-1)\,d}{2} = \frac{2\,s - n\,(n-1)\,d}{2\,n}, \textit{ Ans.}$$

(2) $\frac{4}{5}$. See p. 61, Art. 119, VII.

$$s = \frac{a}{1-r} = \frac{1}{1-\frac{1}{4}} = \frac{4}{5}. \quad \textit{Ans.}$$

(3) Art 114, III.

$$s = \frac{(2\,a + (n-1)\,d)\,n}{2} = \frac{(2 + 16 \times -\frac{5}{4})\,17}{2} = -153. \quad \textit{Ans.}$$

(4) Let the series be $a, a\,r, a\,r^2$, &c., then $s = a + a\,r^2 + a\,r^4 + a\,r^6 + \ldots + a\,r^{2n}$ is the sum of an odd number of terms; and $s' = a^2 + a^2\,r^4 + a^2\,r^8 + a^2\,r^{12} + \ldots + a^2\,r^{4n}$ is the sum of their squares, n being an even number.

$$\therefore \frac{s'}{s} = \frac{a\,(1 + r^4 + r^8 + r^{12} + \ldots + r^{4n})}{1 + r^2 + r^4 + r^6 + \ldots + r^{2n}} = \frac{a\,(1 - r^{4(n+1)}) \div (1 - r^4)}{1 - r^{2(n+1)} \div (1 - r^2)}$$

$$= \frac{a\,(1 + r^{2(n+1)})}{1 + r^2} = a\,(1 - r^2 + r^4) - \ldots + r^{2n};$$

which, since both a and r are integers, must always be a whole number.

9. (1) $1\frac{13}{47}$ day. (2) A will dig $\frac{20}{47}$; B, $\frac{15}{47}$; C, $\frac{12}{47}$.

1863. *July 8th.*

1. (1) $\frac{27 + 4 + 9}{324} = \frac{40}{324} = \frac{10}{81}$, and $\frac{10}{81} - \frac{1}{64} = \frac{559}{5184}$.

(2) $\frac{11}{12} = 1 - \frac{1}{12}$; $\frac{17}{18} = 1 - \frac{1}{18}$; $\frac{23}{24} = 1 - \frac{1}{24}$.

$\therefore \frac{23}{24}$ is the greatest; or see Arts. 66, 67.

2. (1) £1 6*s.* 8*d.* (2) $\frac{105}{8} - \frac{6}{8} = \frac{99}{8}$; $\frac{99}{8}$ of 1*s.* = $2\frac{19}{40}$ of 5*s.*

(3) $\frac{3}{8}$ and $\frac{3}{4}$.

3. $6\frac{1}{4}$ weeks.

4. 2 lbs. of the dearer to 1 lb. of the cheaper.

5. (1) ·06. (2) ·0262 of 10*s.* 6*d.* = ·2751*s.*, and ·38 of 5*s.* = 1·9*s.*, therefore the latter is the greater by 1·6249*s.*

(3) $\frac{33}{50}$, which = ·66.

6. (1) $4x^2$.

(2) $x^2y + x^2z + xy^2 - 2xyz + xz^2 + y^2z + yz^2 - x^3 - y^3 - z^3$. If $y = 0$, the 1st, 3rd, 4th, 6th, 7th, and 9th terms disappear, and the result becomes $x^2z + xz^2 - x^3 - z^3$, which $= (x^2 - z^2)(z - x)$.

7. (1) If $2A = B + C$. $\therefore B = 2A - C$, and $3B = A + C$.

$$\therefore 4B = 3A.$$

(2) $\dfrac{1}{x^2 - \sqrt{2}\,.\,x + 1\,.}$ (3) Art. 100.

(4) $\because a : b :: c : d.$ $\therefore \dfrac{a}{b} = \dfrac{c}{d}.$ $\therefore \dfrac{a+b}{b} = \dfrac{c+d}{d} \ldots (1)$

$$\because \frac{a}{b} = \frac{c}{d} \therefore \frac{a^2}{b^2} = \frac{c^2}{d^2} \therefore \frac{a^2+b^2}{b^2} = \frac{c^2+d^2}{d^2} \ldots\ldots\ldots (2)$$

$$\text{also} \because \frac{a}{b} = \frac{c}{d} \therefore \frac{a^3}{b^3} = \frac{c^3}{d^3} \therefore \frac{a^3+b^3}{b^3} = \frac{c^3+d^3}{d^3} \ldots\ldots (3)$$

$$(3) \div (1) \text{ gives } \frac{a^2+ab+b^2}{b^2} = \frac{c^2+cd+d^2}{d^2} \ldots\ldots (4)$$

$$(4) \div (2) \text{ gives } \frac{a^2+ab+b^2}{a^2+b^2} = \frac{c^2+cd+d^2}{c^2+d^2}.$$

$$\therefore a^2 + ab + b^2 : a^2 + b^2 :: c^2 + cd + d^2 : c^2 + d^2.$$

$\therefore$ (Art. 140) $a^2 + ab + b^2 : c^2 + cd + d^2 :: a^2 + b^2 : c^2 + d^2$.

8. Art. 114, II. $s = \dfrac{n(a+l)}{2}$. (2) $38\frac{1}{2}$. (3) $\dfrac{2}{3}$.

9. (1) $x = 1$. (2) $x = 3$, $y = 2$.

10. Let the first part $= 2x$, the second $3x$, and the third $4x$, then $x = 4$, and the parts are 8, 12, 16. *Ans.*

1864. *January 13th.*

1. (1) £67 10*s.* (2) Arts. 155, 159.

(3) The sum due 15 months hence is £105 8*s.*, and the present value, the rate being 5 per cent., is £99 4*s.*

2. The first gains 1 in 5, or 20 per cent., the other 1 in 6, or $16\frac{2}{3}$.

3. (1) $\dfrac{5}{7} \times \dfrac{4}{9} \times 1\frac{3}{11} \times 6\frac{1}{20} = 2\frac{4}{9}$.

(2) *g. c. m.* $= 45$. $\dfrac{431}{489}$.

(3) $\dfrac{1}{3}$ of $3\frac{1}{2} = \dfrac{7}{6}$. $\dfrac{\frac{2}{35}}{\frac{5}{7\frac{1}{2}}} = \dfrac{3}{35}$.

$$\frac{7}{6} \div \frac{3}{35} = \frac{245}{18} = 13\tfrac{11}{18}.$$

(4) $\dfrac{\frac{1}{2}\left\{\frac{1}{3}-\frac{1}{14}+\frac{6}{7}\left(\frac{1}{2}-\frac{1}{3}\right)\right\}}{1+\dfrac{1}{2+\frac{1}{3}}}=\dfrac{\frac{17}{84}}{\frac{10}{7}}=\dfrac{17}{120}.$

(5) Arts. 55, 56.

4. (1) ·00084. (2) ·045. (3) £1 6*s.* 8*d.* (4) $\frac{9}{80}$.

(5) ·0547074035209 . . . *rem.* 141,828,326,319.

5. (1) Art. 40. (2) Art. 41. (3) $x^2 + 3x - 1$. (4) 0.

(5) $x^2 + 4x + 7 + \dfrac{8}{x-1}$.

6. (1) $x = 1$. (2) 911.

7. (1) Art. 140. (2) $\because \dfrac{a}{b} = \dfrac{c}{d} \therefore \dfrac{a-b}{a+b} = \dfrac{c-d}{c+d}$. . . . . . (A)

also $\dfrac{a^3}{b^3} = \dfrac{c^3}{d^3} \therefore \dfrac{a^3-b^3}{a^3+b^3} = \dfrac{c^3-d^3}{c^3+d^3}$ . . . . (B)

$\therefore (B) \div (A) \dfrac{a^2+ab+b^2}{a^2-ab+b^2} = \dfrac{c^2+cd+d^2}{c^2-cd+d^2}$.

8. (1) Art. 114.

(2) $\frac{1}{3}+\frac{2}{3}+1+\ldots$ to n terms $=\frac{1}{3}\cdot\frac{n\,.\,\overline{n+1}}{2}$ or $\frac{n^2+n}{6}$.

(3) $\frac{1}{3}+\frac{2}{3}+\frac{4}{3}+\ldots$ to n terms $=\frac{1}{3}\cdot\frac{2^n-1}{2-1}=\frac{2^n-1}{3}$.

9. The consumption had diminished in the ratio of 21 : 10.

1864. *June 29th.*

1. (1) P. 9, Art. 20.

(2) Multiply 4567 by 567.

$$\begin{array}{r} 4567 \\ 567 \\ \hline 31969 \\ 27402 \\ 22835 \\ \hline 2589489 \end{array}$$

Here, in multiplying by 7, we multiply by 7 units; in multiplying by 6, we in fact multiply by 60; and in multiplying by 5, we multiply by 500; and by adding the lines thus obtained, we get 567 times 4567 or 2589489.

$$\begin{array}{rrcrrr}
(3)\ 12s.\ 6d. \times & 100 & = & £62 & 10s. & 0d. \\
15s.\ 4d. \times & 100 & = & 76 & 13s. & 4d. \\
16s.\ 8d. \times & 73 & = & 60 & 16s. & 8d. \\
\hline
 & 273 & & £200 & 0s. & 0d.
\end{array}$$

∴ the sum expended weekly is £200.

2. (1) Add $\frac{1}{15}$, $\frac{5}{24}$, $\frac{11}{36}$, and $\frac{7}{40}$.

To find the least common multiple of the denominators.

$$\begin{array}{r|rrrr}
2) & 15, & 24, & 36, & 40 \\
\hline
2) & 15, & 12, & 18, & 20 \\
\hline
2) & 15, & 6, & 9, & 10 \\
\hline
3) & 15, & 3, & 9, & 5 \\
\hline
5) & 5, & 1, & 3, & 5 \\
\hline
 & 1, & 1, & 3, & 1
\end{array}$$

And 2 × 2 × 2 × 3 × 5 × 3 = 360, which is the least common multiple.

∴ the fractions become

$$\frac{24}{360} + \frac{75}{360} + \frac{110}{360} + \frac{63}{360} = \frac{272}{360} = \frac{34}{45}.$$

Again, $\frac{11}{25}$ of $7\frac{11}{12} = \frac{11}{5} \times \frac{19}{12} = \frac{209}{60}$,

and $\frac{34}{45} \div \frac{209}{60} = \frac{34}{3} \times \frac{4}{209} = \frac{136}{627}$. *Ans.*

$$\begin{array}{lrrrr}
(2)\ \frac{5}{6} \text{ of } £75\ 16s.\ 8d. = & £63 & 3s. & 10\frac{2}{3}d. \\
\ \frac{1}{3} \text{ of } 2\ 10s.\ 2d. = & 0 & 16s. & 8\frac{2}{3}d. \\
\hline
 & £64 & 0s. & 7\frac{1}{3}d. & \textit{Ans.}
\end{array}$$

(3) $\frac{£10\ \ 2s.\ 6d.}{£151\ 17s.\ 6d.} = \frac{2430 \text{ pence}}{36450 \text{ pence}} = \frac{1}{15}$. *Ans.*

3. (1) 117 ÷ 625 gives (ciphers being added) ·1872. *Ans.*

(2) $\frac{\cdot 00042}{\cdot 000625} = \frac{42}{625} = \cdot 0672$. *Ans.*

(3) 5656.

(4) ·219.

(5) ·625. £51·2 = £32 is the value required.

4. Arts. 155 and 167. (1) When interest is reckoned only on the original principal, or money lent, it is called *simple interest.*

When the interest at the end of the first period, instead of being paid by the borrower, is retained by him and added on as principal to the former principal, interest being calculated on the new principal for the next period; and this interest, again, instead of being paid, is retained and added on to the last principal for a new principal, and so on; it is called *compound interest.*

(2) Art. 155. $I = \frac{P\,r\,t}{100} =$ £5945 5*s.* 5·96*d.*

(3) Art. 162, or as follows:—In the Bank Stock every £317 gives £10 of interest;

$\therefore$ every £1 of money gives £$\frac{10}{317}$ or £$\frac{30}{951}$ of interest.

In the Consols every £95 gives £3 of interest;

$\therefore$ every £1 gives £$\frac{3}{95}$ or £$\frac{30}{950}$ of interest.

And comparing the fractions, since $\frac{30}{950}$ of any sum is greater than $\frac{30}{951}$ of the same sum, the Consols is the better investment of the two.

5. (1) Since $b > c$, $\therefore b - c$ is positive; and since $a > b - c$, $\therefore a - (b - c)$ is positive. Now in the expression $a - (b - c)$ the whole of b is not to be taken from a, but only the excess of b over c (a positive one).

$\therefore$ If the whole of b be taken from a, we must add c to the remainder thus obtained, or

$$a - (b - c) = a - b + c.$$

(2)
$$\begin{array}{rrrr}
x^3 & +\,4\,x^2\,y & -\,4\,y^3 & \\
3\,x^3 & * & * & -\,5\,x\,y^2 \\
-\,3\,x^3 & -\,x^2\,y & +\,5\,y^3 & -\,x\,y^2 \\
\hline
x^3 & +\,3\,x^2\,y & +\,y^3 & -\,6\,x\,y^2
\end{array}$$

But $(x + y)^3 = x^3 + 3\,x^2\,y + 3\,x\,y^2 + y^3$

$$\begin{array}{rrrr}
x^3 & +\,3\,x^2\,y & -\,6\,x\,y^2 & +\,y^3 \\
\hline
* & * & 9\,x\,y^2 & *
\end{array}$$

$\therefore$ the remainder is $9\,x\,y^2$.

(3)
$$\begin{array}{rrrrr}
x^2 & -\,x\,y & +\,y^2 & & \\
x^2 & +\,x\,y & +\,y^2 & & \\
\hline
x^4 & -\,x^3\,y & +\,x^2\,y^2 & & \\
 & x^3\,y & -\,x^2\,y^2 & +\,x\,y^3 & \\
 & & x^2\,y^2 & -\,x\,y^3 & +\,y^4 \\
\hline
x^4 & & +\,x^2\,y^2 & & +\,y^4
\end{array}$$

And
$$\begin{array}{lllll}
x^4 + & x^2 y^2 + & y^4 & & \\
x^4 + & x^2 y^2 + & y^4 & & \\
\hline
x^8 + & x^6 y^2 + & x^4 y^4 & & \\
 & x^6 y^2 + & x^4 y^4 + & x^2 y^6 & \\
 & & x^4 y^4 + & x^2 y^6 + & y^8 \\
\hline
\multicolumn{5}{l}{x^8 + 2\,x^6 y^2 + 3\,x^4 y^4 + 2\,x^2 y^6 + y^8}
\end{array}$$

Let $x = 1$, and $y = 2$;

Then all the powers of $x = 1$, $y^2 = 4$, $y^4 = 16$, $y^6 = 64$, $y^8 = 256$, and the value is

$$1 + 8 + 48 + 128 + 256 = 441 = 21^2,$$

as it ought to be, since

$$x^4 + x^2 y^2 + y^4 = 21.$$

6. (1)
$$\begin{array}{l}
x - y)\ x^n - y^n\ (x^{n-1} \\
\qquad\quad \underline{x^n - x^{n-1}\, y} \\
\qquad\qquad \overline{\underline{x^{n-1}\, y - y^n}},\ \text{which is the remainder.}
\end{array}$$

But $x^{n-1}\, y - y^n = y\ \overline{(x^{n-1} - y^{n-1})}$,

$\therefore$ the remainder of the quotient becomes

$$y\,\frac{x^{n-1} - y^{n-1}}{x - y}$$

and placing this beside the quotient, x^{n-1}, we have

$$\frac{x^n - y^n}{x - y} = x^{n-1} + y\,\frac{x^{n-1} - y^{n-1}}{x - y}.$$

(2) It can easily be shown by actual division that $x^n - y^n$ is always divisible by $x - y$.

$$\begin{array}{l}
x - y)\ x^n - y^n\ (x^{n-1} + x^{n-2}\, y + x^{n-3}\, y^2 + x^2\, y^{n-3} + x\, y^{n-2} + y^{n-1} \\
\qquad\quad \underline{x^n - x^{n-1}\, y} \\
\qquad\quad x^{n-1}\, y - y^n \\
\qquad\quad \underline{x^{n-1}\, y - x^{n-2}\, y^2} \\
\qquad\qquad x^{n-2}\, y^2 - y^n \\
\qquad\qquad \underline{x^{n-2}\, y^2 - x^{n-3}\, y^3} \\
\qquad\qquad\quad x^{n-3}\, y^3 - y^n
\end{array}$$

Now, in each successive remainder, the index of x is diminished and that of y increased by 1, and the sum of the indices in each term $= n$ $(n - 1 + 1 = n,\ n - 2 + 2 = n,\ n - 3 + 3 = n)$; we shall therefore at length come to a remainder $x^3\, y^{n-3} - y^n$, then

$$\begin{array}{l}
x^3\, y^{n-3} - y^n \\
\underline{x^3\, y^{n-3} - x^2\, y^{n-2}} \\
\quad x^2\, y^{n-2} - y^n \\
\quad \underline{x^2\, y^{n-2} - x\, y^{n-1}} \\
\qquad x\, y^{n-1} - y^n \\
\qquad \underline{x\, y^{n-1} - y^n} \\
\qquad\qquad *
\end{array}$$

(3) $$\frac{x^4 + 4\,x^3 + 8\,x^2 + 41\,x + 36}{x^2 - x + 9} = x^2 + 5\,x + 4.$$

(4) $$\frac{x}{(x-y)(x-z)}+\frac{y}{(y-z)(y-x)}+\frac{z}{(z-x)(z-y)}$$

$$\because \frac{y}{(y-z)(y-x)}=-\frac{y}{(y-z)(x-y)}$$

and $$\frac{z}{(z-x)(z-y)}=\frac{z}{(x-z)(y-z)}$$

$\therefore$ the given expression is equal to

$$\frac{x}{(x-y)(x-z)}-\frac{y}{(y-z)(x-y)}+\frac{z}{(x-z)(y-z)}$$

which $$=\frac{x(y-z)-y(x-z)+z(x-y)}{(x-y)(x-z)(y-z)}$$

$$=\frac{xy-xz-xy+yz+xz-yz}{(x-y)(x-z)(y-z)}$$

$$=0. \quad \textit{Ans.}$$

7. (1) P. 53, Art. 101.

(2) $a^2 b c : a^3 b^2 c^3 :: a^5 b c^6 : \dfrac{a^3 b^2 c^3 \times a^5 b c^6}{a^2 b c}$

$= a b c^2 \times a^5 b c^6 = a^6 b^2 c^8$, which is the fourth proportional.

(3) P. 79, Art. 145.

(4) From absolute equality of ratio we have $3ab : 3ab :: 3cd : 3cd$, and changing the signs of the second and fourth terms, $3ab : -3ab :: 3cd : -3cd$.

But $a : b :: c : d$, multiplying equally, that is, the first terms of both proportions, the second terms, &c., we have

$$3a^2 b : -3ab^2 :: 3c^2 d : -3cd^2$$

But again, from equality of ratio,

$$a^3+b^3 : a^3+b^3 :: c^3+d^3 : c^3+d^3.$$

Adding this proportion to that above,

$$a^3+3a^2 b+b^3 : a^3-3ab^2+b^3 :: c^3+3c^2 d+d^3 : c^3-3cd^2+d^3,$$

which is the proportion required.

8. (1) P. 58, Art. 114, III.

$$S=(2a+\overline{n-1}\,d)\,\frac{n}{2}.$$

(2) Here $a=4$; $d=\frac{1}{3}$; $n=100$.

$\therefore S=(8+99\times\frac{1}{3})\,50=2050.$ *Ans.*

(3) Here $a = 24$; $d = -\frac{1}{2}$; $n = 50$.

$$\therefore S = (48 + 49 \times -\tfrac{1}{2})\, 25 = 587\tfrac{1}{2}. \quad \textit{Ans.}$$

(4) Art. 120, VII. The rth term is $(\frac{2}{3})^{r-1}$.

(5) The sum to n terms is found by Art. 118, II.

(6) If r be a proper fraction and n infinite, $r^n = 0$, and the sum is $a\left(\frac{-1}{r-1}\right)$, or, Art. 120, VII., $\frac{a}{1-r}$.

$$\therefore S = \frac{1}{1-\frac{2}{3}} = \frac{1}{\frac{1}{3}} = 3.$$

9. (1) $\frac{x+1}{15} - \frac{x-9}{16} = \frac{x+9}{5} - \frac{x+1}{4}$.

Clearing of fractions by multiplying each term by 240,

$$\therefore 16\,(x+1) - 15\,(x-9) = 48\,(x+9) - 60\,(x+1);$$

$$\text{Or, } 16\,x + 16 - 15\,x + 135 = 48\,x + 432 - 60\,x - 60.$$

$$\text{Whence } x + 151 = 372 - 12\,x.$$

$$13\,x = 221$$

$$x = 17.$$

(2) $(x+a)\,(x+b) = (x+2\,a)\,(x+2\,b)$;

$$\therefore x^2 + a\,x + b\,x + a\,b = x^2 + 2\,a\,x + 2\,b\,x + 4\,a\,b,$$

$$\text{And } a\,x + b\,x + a\,b = 2\,a\,x + 2\,b\,x + 4\,a\,b,$$

$$0 = a\,x + b\,x + 3\,a\,b;$$

$$\text{Whence } (a+b)\,x = -3\,a\,b,$$

$$\text{And } x = -\frac{3\,a\,b}{a+b}.$$

(3) $15\,x + 16\,y = 108$,

$4\,x + 5\,y = 31$.

Multiply the first equation by 4, and the second by 15, and subtract.

$$\therefore 11\,y = 33,$$

$$\text{And } y = 3;$$

$$\text{But } 4\,x + 5\,y = 31,$$

$$\text{Or, } 4\,x + 15 = 31;$$

$$\therefore 4\,x = 16,$$

$$\text{And } x = 4.$$

10. Let x = total number of men;

y = number in the Prince's Brigade.

Then from the conditions of the question

$$x + 497 = 16\,y \;\ldots\ldots\ldots\; (1)$$

$$x - 9\,(y + 1000) = 233 \;\ldots\ldots\; (2)$$

From (1) $x = 16\,y - 497.$

(2) $x = 233 + 9\,(y + 1000).$

Equating, $16\,y - 497 = 233 + 9\,(y + 1000);$

Or, $16\,y - 497 = 233 + 9\,y + 9000,$

And $16\,y - 9\,y = 9000 + 233 + 497,$

$7\,y = 9730;$

$\therefore\ y = 1390,$

the strength of the Prince's Brigade.

But $x = 16\,y - 497,$

Or $x = 21743$, the total number.

1865. *January* 11*th*.

1. (1) $\dfrac{\text{£}1875}{\text{£}910\ 3s.\ 1\frac{1}{2}d.} = 9s.\ 8\frac{1}{2}d.$ *Ans.*

(2) £57 — $(9s.\ 8\frac{1}{2}d. \times 57) = \text{£}29\ 6s.\ 7\frac{1}{2}d.$ *Ans.*

2. (1) $\dfrac{5}{12} + \dfrac{11}{8} + \dfrac{\frac{3}{2}}{\frac{8}{1}} + \dfrac{\frac{7}{4}}{\frac{13}{8}} = \dfrac{5}{12} + \dfrac{11}{8} + \dfrac{3}{16} + \dfrac{14}{13}$ (*l. c. m.* = 624)

$$= \frac{260 + 858 + 117 + 672}{624} = \frac{1907}{624} = 3\tfrac{35}{624}.$$

$4\frac{1}{4} - 3\frac{35}{624} = 1\frac{1}{4} - \frac{35}{624} = 1\frac{156-35}{624} = 1\frac{121}{624}.$ *Ans,*

(2) ·002. (3) 34·3168.

3. (1) Art. 98. ·2317. (2) 1·6407, &c.

4. (1) Income tax, $\frac{7}{240}$, having been deducted, the net income is £233, $\therefore$ gross income is to net income as $1 : (1 - \frac{7}{240})$, or $\frac{233}{240}$, or $240 : 233$. $\text{£}\frac{240}{3} \times 100 = \text{£}8000$. *Ans.*

(2) $100 : 94\frac{1}{8} :: \text{£}8000 : \text{£}7530$. *Ans.*

5. (1) $x^3 + x^2\,y + x\,y^2 + y^3$. (2) $3\,b$.

6. (1) $\dfrac{1}{3}(5\,x - 2) - \dfrac{1}{7}(21 - 5\,x) = 41 - 4\,x$

$$35\,x - 14 - 63 + 15\,x = 861 - 84\,x$$

$$134\,x = 938,\ \therefore\ x = 7.$$

(2) $\dfrac{6\,x + 21}{11} - \dfrac{8\,x - 2}{7} = \dfrac{4\,x - 3}{5}$

$$210\,x + 735 - 440\,x + 110 = 308\,x - 231$$

$$538\,x = 1076,\ \therefore\ x = 2.$$

(3) $\dfrac{12}{x} + \dfrac{8}{y} = 5;\ \dfrac{27}{x} - \dfrac{12}{y} = 6.$

$$12\,y + 8\,x = 5\,x\,y;\ 27\,y - 12\,x = 6\,x\,y.$$

Multiplying by 9 and 4 respectively,

$$\begin{aligned} 108\,y + 72\,x &= 45\,x\,y \\ 108\,y - 48\,x &= 24\,x\,y \\ \hline 120\,x &= 21\,x\,y \\ 21\,y = 120, \therefore\ y &= 5\tfrac{5}{7}. \\ x &= 3\tfrac{1}{3}. \end{aligned}$$

7. Let x = number of yards.

$$\therefore \frac{120}{x} = \text{price of 1 yard.}$$

$$\therefore \text{ by question } \frac{120}{x+6} + 1 = \frac{120}{x}$$

$$\therefore 126\,x + x^2 = 120\,x + 720$$

(B. A. Course, Art. 178.)* $x^2 + 6\,x = 720$

$$x^2 + 6\,x + (3)^2 = 729$$

$$x + 3 = \pm 27.$$

$$x = 24;\ \frac{120}{x} = 5.$$

8. (1) Art. 101. (2) If $a : b :: c : d$, then (Art. 140) $a : c :: b : d$,

$$\text{And } \frac{a}{c} = \frac{a}{c},\ \frac{a}{c} = \frac{b}{d},\ \therefore\ a\,c = c\,a,\ a\,d = b\,c$$

$$a\,c + a\,d = c\,a + c\,b$$

$$\text{or, } a\,(c + d) = c\,(a + b)$$

$$\therefore \frac{a}{c} = \frac{a+b}{c+d}.$$

(3) $\frac{a}{b} = \frac{c}{d}$ and $\frac{a}{c} = \frac{b}{d}$.

$$\frac{a^2}{c^2} = \frac{a\,b}{c\,d} = \frac{b^2}{d^2} = \frac{l\,a^2}{l\,c^2} = \frac{m\,a\,b}{m\,c\,d} = \frac{n\,b^2}{n\,d^2} = \frac{p\,a^2}{p\,c^2} = \frac{q\,a\,b}{q\,c\,d} = \frac{r\,b^2}{r\,d^2}$$

$$\therefore \frac{l a^2 + m\,a\,b + n\,b^2}{l c^2 + m\,c\,d + n\,d^2} = \frac{p\,a^2 + q\,a\,b + r\,b^2}{p\,c^2 + q\,c\,d + r\,d^2};$$

$$\therefore \frac{p\,a^2 + q\,a\,b + r\,b^2}{l a^2 + m\,a\,b + n\,b^2} = \frac{p\,c^2 + q\,c\,d + r\,d^2}{l c^2 + m\,c\,d + n\,d^2}.$$

9. (1) Art. 114, III. (2) $\frac{17-11}{3} = 2 = d$. Then

$$S = \frac{(2\,a + (n-1)\,d)\,n}{2} = \frac{(22 + 18)\,10}{2} = 200. \quad \textit{Ans.}$$

(3) See p. 62, Ex. 9. The 1st, 2nd, 3rd, and 4th terms may be represented thus, $a,\ a\,r,\ a\,r^2,\ a\,r^3$; but $\frac{a\,r^3}{a} = r^3$, therefore $\frac{24}{3} = 8$, the third power of r, $\therefore\ r = 2$.

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* See foot note to Solution to Question 9, June 1871.

The sum of 8 terms is found (Art. 116, I.) thus:—

$$S = a\,\frac{r^n - 1}{r - 1} = \frac{3}{2}\cdot\frac{256 - 1}{2 - 1} = 382\tfrac{1}{2}. \quad \textit{Ans.}$$

1865. *June 28th.*

1. (1) $\dfrac{9 + 25 + 16 - 50}{60} = 0.$ (2) $\dfrac{\dfrac{5+4}{20}}{\dfrac{5-4}{20}} + \dfrac{\dfrac{4+3}{12}}{\dfrac{4-3}{12}} = \dfrac{9}{1} + \dfrac{7}{1}$

$= 16$ and $16^{\frac{1}{2}} = 4$. *Ans.*

(3) 2500. (4) $\dfrac{\left(\dfrac{33}{16}\right)^3 - 1}{\left(\dfrac{33}{16}\right)^2 + \dfrac{49}{16}} = \dfrac{\dfrac{35937 - 4096}{4096}}{\dfrac{1089 + 784}{256}} = \dfrac{\dfrac{31841}{4096}}{\dfrac{1873}{256}} =$

$\dfrac{31841}{4096} \times \dfrac{256}{1873} = \dfrac{17}{16} = 1\tfrac{1}{16}.$ *Ans.*

2. (1) $\dfrac{28413}{2310} = 12\cdot3.$ (2) 33·1. (3) $\dfrac{4}{2\cdot236 - 1} = \dfrac{4}{1\cdot236} =$

3·2362, &c. (4) $\tfrac{4}{5}$. (5) 6*s.*

$$\begin{array}{r|l}
(6)\ 4 & 3 \\ \hline
12 & 11\cdot75 \\ \hline
20 & \cdot9791\dot{6} \\ \hline
 & \cdot048958\dot{3} \quad \textit{Ans.}
\end{array}$$

3. (1) (B. A. Course, p. 97, Art. 167.) The process of answering this question is the same as would be adopted for finding the Comp. Int. for 4 years at $1\tfrac{1}{4}$ per cent.

$$(1\cdot0125)^4 = 1\cdot509 \times 55 = 57\cdot799.$$

Subtracting the principal we obtain 2·799 = £2 15*s.* $11\tfrac{3}{4}$*d.* *Ans.*

(2) The investor received £9 interest, and by selling at 90 lost £8. ∴ his gain was £1 upon £98.

And 98 : 100 : : 1 : 1·020408 . . . or $1\tfrac{1}{49}$. *Ans.*

4. The three can plant in 1 day, $\dfrac{1}{30} + \dfrac{1}{30} + \dfrac{1}{30} = \dfrac{1}{10}$, but one only does half of $\dfrac{1}{30} = \dfrac{1}{60}$ in 1 day. ∴ the three do in 1 day $\dfrac{1}{30} + \dfrac{1}{30} + \dfrac{1}{60} = \dfrac{1}{12}$. They will plant the whole in 12 days.

5. (1) and (2) Arts. 40 and 46.

(3) $x^2 + x + 1$. *Ans.*

(4) If $a = -(b + c)$, then $\frac{a^2 + b^2}{2} = b^2 + b\,c + \frac{c^2}{2}$.

Now $a\,b = -b^2 - b\,c$

$b\,c = \quad b\,c$

$c\,a = -b\,c - c^2$ and $a\,b + b\,c + c\,a = -b\,c - b^2 - c^2$.

$$\therefore\ \frac{a^2 + b^2}{2} + a\,b + b\,c + c\,a = b^2 + b\,c + \frac{c^2}{2} - b\,c - b^2 - c^2$$

$$= \frac{c^2}{2} - c^2 = -\frac{c^2}{2}.\quad \textit{Ans.}$$

(5) $a^4 - 4\,a^3\,b + 6\,a^2\,b^2 - 4\,a\,b^3 + b^4$.

(6) $$\frac{1}{x+5} - \frac{3}{x + 3 - \frac{6}{x+2}} = \frac{1}{x+5} - \frac{3}{\frac{x^2 + 5\,x + 6 - 6}{x+2}} =$$

$$\frac{x}{x^2 + 5\,x} - \frac{3\,x + 6}{x^2 + 5\,x} = -\frac{2\,x + 6}{x^2 + 5\,x}.\quad \textit{Ans.}$$

(7) Multiplying by com. denom., $(x + 1)\ (x + 2)\ (x + 3)\ (x + 4)$, and cancelling, we obtain for the numerator

$$(x+3)\,(x+4) + (x+1)\,(x+4) + (x+1)\,(x+2) = 3\,(x^2 + 5\,x + 6)$$

$$= 3\,(x + 2)\,(x + 3)$$

then $\frac{3\ (x+2)\,(x+3)}{(x+1)\,(x+2)\,(x+3)\,(x+4)} = \frac{3}{(x+1)\,(x+4)} = \frac{3}{x^2 + 5\,x + 4}$ *Ans.*

6. (1) $\frac{x + 3}{4} - \frac{x - 1}{2} = \frac{1}{3}\left(x + \frac{x}{2} + \frac{3}{2}\right)$

$$3\,x + 9 - 6\,x + 6 = 4\,x + 2\,x + 6$$

$$\therefore\ 9\,x = 9,\ \text{or}\ x = 1.$$

(2) $$\left.\begin{aligned} 3\,x - \quad y &= 3 \\ 5\,x + 7\,y &= 31 \end{aligned}\right\}\qquad \begin{aligned} 15\,x - \ \ 5\,y &= 15 \\ 15\,x + 21\,y &= 93 \\ \hline 26\,y &= 78 \\ y &= \ 3 \ \text{and} \ \therefore\ x = 2. \end{aligned}$$

(3) See p. 73, Ex. 2.

$$x + y + z = \frac{a + b + c}{2}$$

$$\therefore\ x = \frac{a + b + c}{2} - b = \frac{a - b + c}{2}$$

$$y = \frac{a + b + c}{2} - c = \frac{a + b - c}{2}$$

$$z = \frac{a + b - c}{2} - a = \frac{b + c - a}{2}.$$

7. (1) Art. 114, II. $S = n \times$ middle term.

(2) $S = \frac{(2 + (n - 1)\,2)\,n}{2} = n^2$. *Ans.*, and see p. 59, Ex. 4.

(3) Art. 116, I. $S = a\frac{r^n - 1}{r - 1} = 1\,\frac{3^n - 1}{3 - 1} = \frac{3^n - 1}{2}$. *Ans.*

(4) $\frac{9^n - 1}{8}$. Found as in (3).

8. (1) If $a : b :: c : d$, then (Art. 140) $a : c :: b : d$,
and (Art. 146) $m\,a : m\,c :: n\,b : n\,d$
also $p\,a : p\,c :: q\,b : q\,d$.
$\therefore$ (Art. 144) $m\,a + n\,b : m\,c + n\,d :: p\,a + q\,b : p\,c + q\,d$,
and $m\,a + n\,b : p\,a + q\,b :: m\,c + n\,d : p\,c + q\,d$.

(2) $\frac{a^2 + a\,b + b^2}{c^2 + c\,d + d^2} = \frac{a^3 - b^3}{c^3 - d^3} \cdot \frac{d}{b}$. Multiplying by $\frac{b}{d}$

$\frac{b\,(a^2 + a\,b + b^2)}{d\,(c^2 + c\,d + d^2)} = \frac{a^3 - b^3}{c^3 - d^3} \;\therefore\; \frac{b}{d} = \frac{a - b}{c - d}$, and

(Art. 140) $a - b : b :: c - d : d$, and multiplying extremes and means together, $a\,d = b\,c \;\;\therefore\; a : b :: c : d$.

9. Let x = tens' digit, y = units' digit.

$$10\,x + y = 7\,(x + y), \text{ or } 3\,x - 6\,y = 0.$$

$$(10\,x + y) - (10\,y + x) = 27 \text{ or } 9\,x - 9\,y = 27.$$

$$\begin{aligned} 9\,x - 9\,y &= 27 \\ 9\,x - 18\,y &= 0 \\ \hline 9\,y &= 27 \\ y &= 3 \end{aligned}$$

By substitution (in $9\,x - 9\,y = 27$) $x = 6$.
The number is 63.

1866. *January* 10*th*,

1. £419 17*s*. $2\frac{1}{4}$*d*.
2. (1) 10*s*. $3\frac{3}{4}$*d*. (2) ·515625.
3. (1) $\frac{3}{16}$ (2) ·1875.
4. £1303 6*s*. 8*d*.
5. (1) 62573. (2) $6\frac{7}{15}$.
6. (1) $\frac{a^2\,b^2}{a^4 - b^4}$. (2) $\frac{4}{15}\,b^2$.
7. (1) Art. 145. This question was given at *Matriculation* 1838-9-41, &c.

(2) If $a : b :: c : d$

$$\frac{a}{b} = \frac{c}{d} = \frac{a^2}{a\,b} = \frac{c^2}{c\,d} = \frac{a^2 + c^2}{a\,b + c\,d}$$

$$= \frac{a\,b}{b^2} = \frac{c\,d}{d^2} = \frac{a\,b + c\,d}{b^2 + d^2}$$

$$\text{and } \because \frac{a^2 + c^2}{a\,b + c\,d} = \frac{a\,b + c\,d}{b^2 + d^2}$$

$$\therefore (a^2 + c^2)\,(b^2 + d^2) = (a\,b + c\,d)^2. \quad \textit{Ans.}$$

8. (1) Art. 117. (2) $\frac{3}{11}$. (3) See p. 59, Ex. 2. The means are 15, 17, 19, 21.

9. (1) $x = 3$. (2) $x = 3, y = 2$.

10. Tea 2*s.* 6*d.* per lb., sugar 8*d.* per lb.

1866. *June 27th.*

1. (1) $\frac{1}{4}$. (2) 101 tons, 5 cwt., 2 qrs., 19 lbs.

2. (1) ·0810089020771513, &c. (2) 7·194, &c.
(3) $\frac{13}{20}$. (4) ·4875.

3. (1) £33 6*s.* 8*d.* (2) 75.

4. P. 48, Quest. 15. The conjunction between 2 and 3 o'clock is at $10\frac{10}{11}$ min. By the question, min. hand moves 6 degrees, and hour hand $\frac{1}{2}$ degree, in 1 minute of time $\therefore$ 1 minute after conjunction they are $5\frac{1}{2}$ degrees apart. Then $5\frac{1}{2}° : 90° :: 1\text{ m.} : 16\frac{4}{11}$ m., and $10\frac{10}{11} + 16\frac{4}{11} = 27\frac{3}{11}$ min. past 2 o'clock. *Ans.*

5. (1) $8\,(x^4 + y^4)$. (2) $\frac{x - 4}{x - 8}$. (3) $a + \sqrt{a^2 - b^2}$, or 0.

6. (1) $x = 2$. (2) $x = \frac{1}{2}a;\ y = -\,b$.

$$(3)\quad a\,x + b\,y + c\,z = 0 \quad \text{(I.)}$$
$$a^1 x + b^1 y + c^1 z = 0 \quad \text{(II.)}$$

Multiplying I. by b^1 and II. by b

$$\therefore a\,b^1 x + b\,b^1 y + c\,b^1 z = 0$$
$$\underline{a^1 b\,x + b\,b^1 y + c^1 b\,z = 0}$$
$$\therefore (a\,b^1 - a\,b)\,x + (c\,b^1 - c^1 b)\,z = 0$$
$$\therefore (a\,b^1 - a^1 b)\,x = (c^1 b - c\,b^1)\,z$$
$$\therefore \frac{x}{b\,c^1 - b^1 c} = \frac{z}{a\,b^1 - a^1 b}$$

Multiply the first equation by c^1, and the second by c, and subtracting, we find

$$\frac{x}{b\,c^1 - b^1 c} = \frac{y}{c\,a^1 - c^1 a}$$

$$\therefore \frac{x}{b\,c^1 - b^1 c} = \frac{y}{c\,a^1 - c^1 a} = \frac{z}{a\,b^1 - a^1 b}.$$

7. (1) P. 60, Art. 117. $a\,r$, and $a\,r^{n-2}$, are the second and last but one terms, both given. Now $a\,r^{n-2}.\,r = a\,r^{n-1}$, the *la*

term required; and $\frac{a\,r^{n-1}}{a\,r^{n-2}} = r$, the ratio required; also $\frac{a\,r}{r} = a$, the first term required. Hence by I. $S = a\,\frac{r^n - 1}{r - 1}$. *Ans.*

(2) P. 58 (I.) $n = 934$; then by II., $S = 1867^2 = 3485689$.
(3) $2\,(1 - (\tfrac{1}{2})^n)$. (See Ex. 2, p. 61.)
(4) $2^n - 1$. (See Ex. 1, p. 61.)

8. (1) See Quest. 5, p. 82. If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$, then $\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} = \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b}$; or $\frac{a}{d} = \frac{a^3}{b^3}$ $\therefore a : d :: a^3 : b^3$, which explains algebraically what is meant by Triplicate Ratio. *Euclid*, Book V., def. 11.

(2) 4 A's = 5 B's steps,
$1 = 1\tfrac{1}{5}$ $\therefore$ 6 of A's $= 6 \times 1\tfrac{1}{5} = 7\tfrac{1}{5}$.
Again 5 B's = 4 A's.
$1 = \tfrac{4}{5}$ $\therefore$ 7 of B's $= 7 \times \tfrac{4}{5} = 5\tfrac{3}{5}$.
A is the quicker of the two.

9. A butt of A, £40. A butt of B, £80.
10. $4\tfrac{1}{4}$ square yards. Cost, 17*s.*

1867. *January* 16*th.*

1. £348 14*s.* $9\tfrac{1}{4}$*d.*
2. (1) $3\tfrac{67}{96}$. (2) $5\tfrac{1}{7}$.
3. £11 13*s.* 4*d.*
4. 28125.
5 (1) 4·92. (2) 55·9. (3) 27·5028.
6. (1) By actual multiplication $(a-b)^3 + (b-c)^3 + (c-a)^3 =$

$$\begin{aligned} &a^3 - 3\,a^2 b + 3\,a\,b^2 - b^3 \\ &+ b^3 - 3\,b^2 c + 3\,b\,c^2 - c^3 \\ &+ c^3 - 3\,c^2 a + 3\,c\,a^2 - a^3; \end{aligned}$$

Similarly $3\,(a-b)\,(b-c)\,(c-a) =$

$$\begin{aligned} 3\,a\,b\,c &- 3\,a^2 b + 3\,a\,b^2 \\ &- 3\,b^2 c + 3\,b\,c^2 \\ &- 3\,c^2 a + 3\,c\,a^2 - \quad a\,b\,c; \end{aligned}$$

and the two sums become identical.

(2) $a^2\,(a+b) - a\,x\,(a+b) + x^2\,(a+b)$. (3) $a^2 + a\,x + x^2$.

7. $\frac{2\,x^4 - 9\,x^3 - 14\,x + 3}{3\,x^4 - 14\,x^3 - 9\,x + 2} = \frac{2\,x^2 + x + 3}{3\,x^2 + x + 2}$. $g.\,c.\,m. = x^2 - 5 \quad + 1$.
8. (1) $x = 16$. (2) $x = 6$, $y = 12$.
9. (1) Art. 114. (2) Ex. 4, p. 59. (3) The common ratio is 2, and the series 1, 2, 4, 8, 16, 32, 64, 128.

10. 8 lbs. of the better sort, 32 lbs. of the worse.

1867. *June 26th.*

1. 18*s.*

2. (1) $1\frac{47}{63}$. (2) £6 5*s.* $2\frac{1}{4}$*d.*

3. (1) $10\frac{3}{8}$. (2) 1·65.

4. £231 10*s.* $0\frac{33}{100}$*d.*

5. (1) 27·02. (2) $2\frac{3}{23}$

6. (1) $$\frac{x+\frac{6}{x-5}}{x+6+\frac{24}{x-5}}=\frac{x^2-5x+6}{x^2+x-6}=\frac{(x-2)(x-3)}{(x-2)(x+3)}=\frac{x-3}{x+3}.\ \textit{Ans.}$$

(2) $$\frac{1}{1+\frac{x^2}{1+x}}+\frac{1}{1-x}=\frac{1+x}{1+x+x^2}+\frac{1}{1-x}=\frac{1-x^2+1+x+x^2}{1-x^3}$$

$$=\frac{2+x}{1-x^3}.\ \textit{Ans.}$$

(3) If $x=2$, $$\frac{1}{1+\frac{x^2}{1+x}}=\frac{1}{\frac{7}{3}}=\frac{3}{7};\ \frac{1}{1-x}=\frac{1}{-1}=-1$$

$$\frac{3}{7}-1=\frac{3-7}{7}=-\frac{4}{7}.\ \textit{Ans.}$$

7. Art. 101. (1) Art. 144.

(2) From Art. 145. $$\frac{a-c}{a+c}=\frac{b-d}{b+d}\ldots\text{(I.)}$$

and $$\frac{a^3}{c^3}=\frac{b^3}{d^3}\ \therefore\ \frac{a^3-c^3}{a^3+c^3}=\frac{b^3-d^3}{b^3+d^3}.\ \text{(II.)}$$

Divide (II.) by (I.)

$$\frac{\frac{a^3-c^3}{a-c}}{\frac{a^3+c^3}{a+c}}=\frac{\frac{b^3-d^3}{b-d}}{\frac{b^3+d^3}{b+d}}$$

or $$\frac{a^2+ac+c^2}{a^2-ac+c^2}=\frac{b^2+bd+d^2}{b^2-bd+d^2}.$$

8. (1) Art. 114. (2) See p. 62 (9), the three means are $13\frac{1}{2}$, 36, and 96.

9. (1) $x=1$.

(2) $$\begin{array}{rcll} 3y-2x & = & 5xy \ldots\ldots & \text{I.} \\ 5y+3x & = & 21xy \ldots\ldots & \text{II.} \\ \hline 9y-6x & = & 15xy & \text{I} \times 3 \\ 10y+6x & = & 42xy & \text{II} \times 2 \\ \hline 19y & = & 57xy\ \therefore & \end{array}$$

$\frac{19\,y}{57\,y} = x = \frac{1}{3}$ and substituting this value of x

in I, $y = \frac{1}{2}$.

10. Since £40,000 is 4 per cent. on $\frac{4}{5}$ of the capital

$\frac{4}{5}$ of the capital is £1,000,000,

∴ the whole capital is £1,250,000.

Also 5 per cent on $\frac{1}{5}$ of the capital is £12,500, which, together with the £40,000, forms 41 per cent. of the receipts.

∴ 41 per cent. of the whole receipts is £52,500,

∴ the whole receipts are $\frac{5250000}{41}$ or £128048·$\dot{7}804\dot{8}$,

or £128,048 15*s.* 7¼*d.* (7$\frac{13}{41}$ exactly.)

1868. *January* 15*th*.

1. (1) $\frac{1}{12}$. (2) 769240.
2. (1) 17·8609375. (2) 14·697.
3. £13 15*s.* 7½*d.*
4. (1) £15 15*s.* 3*d.* (2) £1225.
5. (1) ·94868. (2) 19·48558. (3) 443329.
6. (1) $a^2 + b^2 + c^2 - ab - ac - bc$. (2) 0.
7. $\frac{4\,x^2 - 3\,x - 4}{5\,x^2 - 4\,x - 3}$.
8. (1) $x = 5$.

(2) $\frac{x}{a+b} + \frac{y}{a-b} = 2 \ldots\ldots \text{(I.)}$

$$b\left(\frac{1}{x} + \frac{1}{y}\right) = a\left(\frac{1}{y} - \frac{1}{x}\right) . \text{(II.)}$$

From (II.) $\frac{b}{x} + \frac{b}{y} = \frac{a}{y} - \frac{a}{x}$

$$\therefore \frac{b+a}{x} = \frac{a-b}{y}$$

$$\therefore \frac{x}{a+b} = \frac{y}{a-b}.$$

Substituting in (I.)

gives $\left.\begin{array}{l} x = a + b \\ y = a - b \end{array}\right\}$ *Ans.*

9. (1) Art. 113, p. 58, and see questions 2 and 6, p. 59.

(2) Two arith. means between a and c,

com. diff. $= \frac{c-a}{3}$; and the four terms are $a, \frac{2a+c}{3}, \frac{a+2c}{3}$ and c.

Two geom. means between a and c; then $r = \sqrt[3]{\frac{c}{a}}$ and the four terms are $a, \sqrt[3]{a^2 c}, \sqrt[3]{a c^2}, c$.

Product of arith. means is $\frac{2a^2 + 5ac + 2c^2}{9}$; of geom. means, $\sqrt[3]{a^3 c^3}$ or $a c$. If $a = c$ the two products are equal: but suppose that a is not equal to c, but $= c - k$; then $a c = c^2 - k c$, which is the product of the geom. means. $2a^2 + 5ac + 2c^2 = 9c^2 - 9kc + 2k^2$, and the product of the arithmetical means is $c^2 - kc + \frac{2k^2}{9}$; which is greater than the product of the geom. means. If we had supposed a to be greater than c, we should have had a similar result.

10. 15 galls. of water.

1868. *July* 1st.

1. £57 3*s.* 0*d.*
2. (1) 34*s.* (2) 4·08.
3. £27 1*s.* 4*d.*
4. £231 10*s.* $0\frac{33}{100}$*d.*
5. (1) ·0316. (2) $2\frac{3}{23}$.
6. (1) $x - 1$. (2) *g. c. m.* $x^2 + 5x + 6$, and the fraction simplified $\frac{x+1}{x}$. (3) 0.

7. (1) Art. 103, p. 53.

(2) $$\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$$

$$\therefore \frac{a^3}{b^3} = \frac{c^3}{d^3} = \left(\frac{a+c}{b+d}\right)^3$$

$$\therefore \frac{a^3+c^3}{b^3+d^3} = \left(\frac{a+c}{b+d}\right)^3$$

8. (1) Art. 113. The difference between the sum of n terms, and of $\overline{n+1}$ terms is $a + n d$ (which is the $\overline{n+1}$th term): and the difference between the sum of n terms and of $\overline{n-1}$ terms is $a + \overline{n-1}\, d$ (which is the nth term). Both these differences are known, and the latter subtracted from the former leaves d the common difference.

(2) First term is x^n, ratio $\frac{y}{x}$ number of terms $n + 1$.

$$\therefore \quad S = x^n . \frac{1 - \left(\frac{y}{x}\right)^{n+1}}{1 - \frac{y}{x}} = x^{n+1} . \frac{1 - \left(\frac{y}{x}\right)^{n+1}}{x - y} = \frac{x^{n+1} - y^{n+1}}{x - y}$$

(3) nth term is $1 + \frac{1}{2}(n - 1) = \frac{n + 1}{2}$. $S = \left(\frac{n + 1}{2} + 1\right)\frac{n}{2}$

$$= \frac{n^2 + 3\,n}{4}. \quad \textit{Ans.}$$

9. Let x be the value of the stock.

Then by the question

$$\frac{x}{4} + 500 = \left(\frac{3x}{4} - 500\right) \times 2$$

$$x + 2000 = 6\,x - 4000$$

$$\therefore\ 5\,x = 6000$$

$$\therefore\ \quad x = 1200.$$

10. (1) $x = 1$. (2) $x = 1$; $y = 1$.

1869. *January* 13*th*.

1. £3 9*s*. 1½*d*.
2. (1) 1*s*. 5½*d*. (2) 4*s*. 11¼*d*.
3. (1) $\frac{22}{81}$. (2) $326\frac{2}{5}$.
4. 10 years.
5. (1) 437·962. (2) 22·516.
6. (1) *g. c. m.*, $x - 4$, and the fraction simplified becomes $\frac{x^2 - 6\,x + 2}{x^2 - 5\,x + 3}$.

(2) $\frac{2\,(x - 1}{(x - 2)\,(x - 3)} - \frac{x - 2}{(x - 1)\,(x - 3)} - \frac{x - 3}{(x - 1)\,(x - 2}$)

$$= \frac{2\,(x - 1)^2 - (x - 2)^2 - (x - 3)^2}{(x - 1)\,(x - 2)\,(x - 3)}$$

$$= \frac{2\,x^2 - 4x + 2 - x^2 + 4\,x - 4 - x^2 + 6\,x - 9}{(x - 1)\,(x - 2)\,(x - 3)} = \frac{6\,x - 11}{(x - 1)\,(x - 2)(x - 3)}$$

$$= \frac{6\,x - 11}{x^3 - 6\,x^2 + 11\,x - 6}. \quad \textit{Ans.}$$

7. (1) P. 53, Art. 101.

(2) $\frac{a^2 + a\,b + b^2}{a^2 - a\,b + b^2} = \frac{c^2 + c\,d + d^2}{c^2 - c\,d + d^2}$

$$\therefore \frac{a^2 + b^2}{a\,b} = \frac{c^2 + d^2}{c\,d}$$

$$\text{or } \frac{a}{b} + \frac{b}{a} = \frac{c}{d} + \frac{d}{c}$$

$$\frac{a}{b} - \frac{d}{c} = \frac{c}{d} - \frac{}{a}$$

$$\frac{ac - bd}{bc} = \frac{ac - bd}{ad}$$

$$\therefore ad = bc \; \therefore \frac{a}{b} = \frac{c}{d}$$

$$\therefore a : b :: c : d.$$

$$\text{Also } \frac{a}{b} - \frac{c}{d} = \frac{d}{c} - \frac{b}{a}$$

$$\frac{ad - bc}{bd} = \frac{ad - bc}{ac}$$

$$\therefore ac = bd$$

$$\therefore \frac{a}{b} = \frac{d}{c}$$

$$\therefore a : b :: d : c.$$

8. (1) P. 58, Art. 112. (2). P. 58, Formula (III.)
Substituting, according to the terms of the question,

$$100\,a = \frac{(2\,a + (n-1)\,2\,a)\,n}{2}$$

$$200\,a = 2\,n\,a + 2\,n^2 a - 2\,n\,a$$

$$n^2 = 100$$

$$n = 10. \quad \textit{Ans.}$$

9. (1) $x = 2$.

(2) First $(x+y)(a-b) + (x-y)(a+b) = 2\,(a+b)(a-b)$
or $a\,x + a\,y - b\,x - b\,y + a\,x - a\,y + b\,x - b\,y = 2\,(a^2 - b^2)$

$$\begin{aligned} \text{Reducing, } a\,x - b\,y &= a^2 - b^2 \\ a\,x + b\,y &= a^2 + b^2 \\ \hline 2\,a\,x \quad &= 2a^2 \\ x \quad &= a \text{ and } \therefore y = b. \quad \textit{Ans.} \end{aligned}$$

10. Let x = distance in miles from A to B
y = number of hours walked simultaneously.

By the question, traveller from A, $\frac{x}{2} - 2\frac{3}{4} = 3\,y + \frac{1}{4}$.

B, $\frac{x}{2} + 2 = 3\frac{3}{4}\,y - 1$.

$$\begin{aligned} 2\,x - 12 &= 12\,y \\ 2\,x + \;\; 9 &= 15\,y \\ \hline 21 &= \;\; 3\,y \\ y &= \;\; 7 \text{ hours } \therefore x = 48 \text{ miles.} \end{aligned}$$

First traveller walked $7\frac{1}{4}$ hours, 3 m. per h. = $21\frac{3}{4}$
Second „ 7 „ $3\frac{3}{4}$ „ $\underline{26\frac{1}{4}}$

1869. *June 30th.*

1. (1) 2260. (2) $\frac{1}{7}$. (3) 1.
2. ·864.
3. 6·37*d.* nearly.
4. (1) £97 5*s.* 11$\frac{13}{37}$*d.* (2) £21 9*s.* 9$\frac{21}{47}$*d.*
5. (1) 6. (2) 5·6692.
6. (1) $x^2 + 2x - 1$. (2) Clearing the fractions and dividing by *g. c. m.* $x + 1$, we get $\frac{x^3 - x^2}{x^2 + 1}$.

$$(3)\ \frac{x^3 - 3x + 2}{2x^3 - 3x^2 + 1} = \frac{(x^2 - 2x + 1)(x + 2)}{(x^2 - 2x + 1)(2x + 1)} = \frac{x + 2}{2x + 1}. \quad \textit{Ans.}$$

7. (1) P. 79, Art. 145. (2) $\frac{a}{b} = \frac{c}{d}$

$$\frac{a^2}{b^2} = \frac{c^2}{d^2} = \frac{ac}{bd}$$

$$\therefore \left(\frac{a}{b}\right)^2 + \left(\frac{c}{d}\right)^2 = 2\frac{ac}{bd}. \quad \textit{Ans.}$$

8. (1) P. 58, Art. 115. (2) a, b, c, d, are in arith. prog.

$$\frac{a + c}{2} = b \ \therefore \frac{a^2 + 2ac + c^2}{4} = b^2$$

$$\therefore a^2 + c^2 = 4b^2 - 2ac$$

$$\frac{b + d}{2} = c \ \therefore \frac{b^2 + d^2}{2} = 2c^2 - bd$$

$$\therefore \frac{b^2 + d^2}{2} + a^2 + c^2 = 4b^2 + 2c^2 - 2ac - bd$$

$$\text{but } \because b + c = a + d$$

$$\therefore (b + c)^2 = (a + d)(b + c)$$

$$b^2 + c^2 = ab + ac + bd + cd - 2bc$$

$$\text{now } b + d = 2c$$

$$\therefore b^2 + c^2 = ab + ac + bd + cd - b^2 - bd$$

$$= ab + ac + cd - b^2$$

$$\therefore 2b^2 + c^2 = ab + ac + cd$$

$$\therefore 2b^2 + c^2 - ac = ab + cd$$

$$\therefore 4b^2 + 2c^2 - 2ac = 2(ab + cd)$$

$$\frac{b^2 + d^2}{2} + a^2 + c^2 = 2(ab + cd) - bd. \quad \textit{Ans.}$$

9. (1) $x = 1$. (2) $x = 2$; $y = 2$.
10. A walks 58$\frac{2}{3}$ yds. in 1 min.

At 10 min. past 2, (A) was 586$\frac{2}{3}$ yards from station A.

Of the remaining 1173$\frac{1}{3}$ yards,

(B) walks $\frac{2}{3}$, 782$\frac{2}{9}$ „

(A) walks $\frac{1}{3}$, 391$\frac{1}{9}$ „

they meet 391$\frac{1}{9}$ + 586$\frac{2}{3}$ or 977$\frac{7}{9}$ yards from A's starting place. *Ans.*

$$\text{or } \frac{a}{b} + \frac{b}{a} = \frac{c}{d} + \frac{d}{c}$$

$$\frac{a}{b} - \frac{d}{c} = \frac{c}{d} - \frac{}{a}$$

$$\frac{ac - bd}{bc} = \frac{ac - bd}{ad}$$

$$\therefore ad = bc \therefore \frac{a}{b} = \frac{c}{d}$$

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$$\text{Also } \frac{a}{b} - \frac{c}{d} = \frac{d}{c} - \frac{b}{a}$$

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$$n = 10. \quad \textit{Ans.}$$

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or $a\,x + a\,y - b\,x - b\,y + a\,x - a\,y + b\,x - b\,y = 2\,(a^2 - b^2)$

Reducing, $a\,x - b\,y = a^2 - b^2$

$$\begin{aligned} a\,x + b\,y &= a^2 + b^2 \\ \hline 2\,a\,x &= 2a^2 \\ x &= a \text{ and } \therefore y = b. \quad \textit{Ans.} \end{aligned}$$

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By the question, traveller from A, $\frac{x}{2} - 2\frac{3}{4} = 3\,y + \frac{1}{4}$.

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1869. *June 30th.*

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5. (1) 6. (2) 5·6692.

6. (1) $x^2 + 2x - 1$. (2) Clearing the fractions and dividing by *g. c. m.* $x + 1$, we get $\frac{x^3 - x^2}{x^2 + 1}$.

(3) $\frac{x^3 - 3x + 2}{2x^3 - 3x^2 + 1} = \frac{(x^2 - 2x + 1)(x + 2)}{(x^2 - 2x + 1)(2x + 1)} = \frac{x + 2}{2x + 1}$. *Ans.*

7. (1) P. 79, Art. 145. (2) $\frac{a}{b} = \frac{c}{d}$

$$\frac{a^2}{b^2} = \frac{c^2}{d^2} = \frac{ac}{bd}.$$

$$\therefore \left(\frac{a}{b}\right)^2 + \left(\frac{c}{d}\right)^2 = 2\frac{ac}{bd}. \quad \textit{Ans.}$$

8. (1) P. 58, Art. 115. (2) a, b, c, d, are in arith. prog.

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$$\therefore a^2 + c^2 = 4b^2 - 2ac$$

$$\frac{b + d}{2} = c \;\therefore\; \frac{b^2 + d^2}{2} = 2c^2 - bd$$

$$\therefore \frac{b^2 + d^2}{2} + a^2 + c^2 = 4b^2 + 2c^2 - 2ac - bd$$

$$\text{but} \because b + c = a + d$$

$$\therefore (b + c)^2 = (a + d)(b + c)$$

$$b^2 + c^2 = ab + ac + bd + cd - 2bc$$

$$\text{now } b + d = 2c$$

$$\therefore b^2 + c^2 = ab + ac + bd + cd - b^2 - bd$$

$$= ab + ac + cd - b^2$$

$$\therefore 2b^2 + c^2 = ab + ac + cd$$

$$\therefore 2b^2 + c^2 - ac = ab + cd$$

$$\therefore 4b^2 + 2c^2 - 2ac = 2(ab + cd)$$

$$\frac{b^2 + d^2}{2} + a^2 + c^2 = 2(ab + cd) - bd. \quad \textit{Ans.}$$

9. (1) $x = 1$. (2) $x = 2$; $y = 2$.

10. A walks 58$\frac{2}{3}$ yds. in 1 min.
At 10 min. past 2, (A) was 586$\frac{2}{3}$ yards from station A.
Of the remaining 1173$\frac{1}{3}$ yards,
(B) walks $\frac{2}{3}$, 782$\frac{2}{9}$ „
(A) walks $\frac{1}{3}$, 391$\frac{1}{9}$ „
they meet 391$\frac{1}{9}$ + 586$\frac{2}{3}$ or 977$\frac{7}{9}$ yards from A's starting place. *Ans.*

1870. *January 12th.*

1. (1) $\frac{19}{280}$. (2) 7. (3) ·0002929678. Art. 93.
2. (1) ·00015625. (2) 1 cwt. 1 qr. 20 lbs.
3. (1) £105 8*s.* 4*d.* (2) £104 7*s.* 3*d.*
4. (1) £500. (2) £541 13*s.* 4*d.*
5. (1) 100372. (2) ·1715. (3) P. 52, Art. 99 (3).
6. (1) Arts. 23 and 25. (2) $a + 2b + 3c$.
7. (1) $1 + \frac{b}{a}$. (2) $g.\ c.\ m. = x^2 - 3x + 2$ and fraction in lowest terms is $\frac{x}{x+3}$.
8. (1) $x = 13$. (2) $x = -\frac{a}{a+b-1}$; $y = \frac{b}{a+b-1}$.
9. (1) P. 58, Art. 114. (2) P. 61. II. $S = a\frac{1-r^n}{1-r} =$

$$\frac{3}{2}\ \frac{1-\frac{1}{3}^n}{1-\frac{1}{3}} = \frac{\frac{3}{2}}{\frac{2}{3}} \times \left(1 - (\tfrac{1}{3})^n\right) = \tfrac{9}{4}\left(1 - (\tfrac{1}{3})^n\right).\ \textit{Ans.}$$

(3) P. 61, VII. $S = \frac{a}{1-r} = \frac{n+1}{n}\left(\frac{1}{1+\frac{n}{n+1}}\right) = \frac{(n+1)^2}{2n^2+n}$.

10. (1) P. 79, Art. 144. (2) Arts. 145 and 147.

1870. *June 29th.*

1. £902777 15*s.* $6\frac{2}{3}$*d.*
2. (1) $3\frac{3}{16}$. (2) $\frac{689}{11000}$.
3. (1) ·01357. (2) $16\frac{3}{7} = 16{\cdot}4285 \ldots.$
4. (1) P. 85. Arts. 155 and 159. (2) £114 4*s.* $2\frac{2}{5}$*d.*
5. A, £20; B, £21; C, £25.
6. (1) $a^3+b^3-c^3-a^2b+a^2c-ab^2-ac^2+b^2c-bc^2+6abc$.

(2) $x^2 - 4x - 1$. (3) $\frac{a^2+2ab+2b^2}{a^2+2ab+b^2} = 1$.

7. (1) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then $\frac{a}{b} = \frac{a+c+e}{b+d+f}$. For let $\frac{a}{b} = x = \frac{c}{d} = \frac{e}{f}$, then $a = bx$, $c = dx$, and $e = fx$, and $a + c + e =$

$$bx + dx + fx = (b + d + f)\,x$$

$$\therefore\ x \text{ or } \frac{a}{b} = \frac{a+c+e}{b+d+f} \text{ or}$$

$$a : b :: a + c + e :: b + d + f.$$

8. (1) 125000250000. (2) P. 60, Art. 117.

9. (1) $(6 - x)(1 + 2x) + (5 + x) 3x = (x + 1)^2 - x$

$$6 + 11x - 2x^2 + 15x + 3x^2 = x^2 + 2x + 1 - x$$

$$25x = -5 \quad \therefore x = -\tfrac{1}{5}.$$

(2) $x = 5$; $y = 5$.

10. Let x = amount of increase in debt, then $3x$ = debt before the war.

In 10 years £20,000,000 was paid off.

$$3x \times \tfrac{6}{100} = (4x - 20,000,000) \times \tfrac{5}{100}$$

$$x = 50,000,000. \quad \textit{Ans.}$$

1871. *January* 11*th*.

1. (1) $\frac{3}{8}$. (2) ·0000152.

2. (1) Arts. 47 and 48. (2) Art. 39. (3) $2^2 \times 7 \times 13 = 364$; $2^3 \times 3^2 \times 5 \times 7 = 2520$; $3^2 \times 5 \times 11^2 = 5445$.

3. (1) £3 6*s.* 4¼*d.* (2) ·2988.

4. (1) $\frac{121}{123}$. (2) 209·25.

5. 14479673·9, &c., galls.

6. (1) a^4. (2) $\dfrac{x^2 - y^2}{x^{\frac{1}{2}} - y^{\frac{1}{2}}} = x^{\frac{3}{2}} + x y^{\frac{1}{2}} + x^{\frac{1}{2}} y + y^{\frac{3}{2}}$. *Ans.*

(3) $\dfrac{4x^4 + 1}{2x^2 + 2x + 1} = 2x^2 - 2x + 1$. *Ans.*

7. (1) $\dfrac{1+x-4x^2-4x^3}{1-7x^2-6x^3} = \dfrac{(1+3x+2x^2)(1-2x)}{(1+3x+2x^2)(1-3x)} = \dfrac{1-2x}{1-3x}$. *Ans.*

(2) $\dfrac{x^3}{(x-1)(x-2)} + \dfrac{1}{x-1} - \dfrac{8}{x-2} = \dfrac{x^3+x-2-8x+8}{(x-1)(x-2)} =$

$$\frac{x^3-7x+6}{x^2-3x+2} = x+3. \quad \textit{Ans.}$$

8. (1) P. 53, Art. 101. (2) $\dfrac{a}{b} = \dfrac{c}{d} \quad \therefore ad = bc$, and $a^2 d^2 =$

$$b^2 c^2 \quad \therefore \frac{ad}{c^2} = \frac{b^2}{ad}$$

$\dfrac{a^2 d}{c^2 b} = \dfrac{b^2 c}{d^2 a} \quad \therefore \dfrac{a^2}{b} : \dfrac{c^2}{d} :: \dfrac{b^2}{a} : \dfrac{d^2}{c}$. or $\dfrac{a^2}{b}$ is to $\dfrac{c^2}{d}$ inversely as $\dfrac{a}{b^2} : \dfrac{c}{d^2}$.

(3) $a : a\sqrt{b} :: a\sqrt{b} : ab$; and ab is the third proportional required.

(4) A mean proportional between a and b is $\sqrt{ab}$, and a mean proportional between $\dfrac{p+q}{p-q} a^2 b$ and $\dfrac{p-q}{p+q} a b^2$ is $\sqrt{a^3 b^3} = ab\sqrt{ab}$.

9. (1) $S = 0$. (2) $1\frac{1}{7}$.

(3) P. 61, Art. 120. The "sum to infinity" means that it can be made to differ as little as we please from $\frac{a}{1-r}$. (4) When r is a fraction less than unity.

10. (1) $x = 2$. (2) $x = \frac{5}{7}a$.

(3) By question $2x + 3y = xy$ (1)

$$10x - 9y = xy \quad (2)$$

(2) − (1). Then $8x - 12y = 0$ and $x = \frac{3}{2}y$

and $xy = \frac{3}{2}y \, . \, y$.

$\therefore \; 6y = \frac{3}{2}y^2; \; y^2 = 4y$ and $y = 4$.

Then substituting the value of y in (1)

$$2x + 12 = 4x$$

$$2x = 12, \text{ and } x = 6.$$

1871. *June 28th.*

1. 2415 times.
2. (1) $3\frac{1}{3}$. (2) $3\frac{9}{17}$.
3. (1) $\frac{17}{63}$. (2) $\frac{11}{30}$.
4. (1) ·3183. (2) 808.
5. (1) 180000·0156. (2) Reducing to com. denom. we obtain

$$\frac{5 - 6\sqrt{5} + 9 - (5 + 6\sqrt{5} + 9)}{5 - 3\sqrt{5} + 3\sqrt{5} - 9} = \frac{-12\sqrt{5}}{-4} = 3\sqrt{5} = \sqrt{45} = 6{\cdot}7082.$$

6. The man can do $\frac{1}{77}$ in 1 hour, a boy $\frac{1}{121}$;

Man and boy $\frac{1}{77} + \frac{1}{121}$ in 1 hour.

$$\frac{1}{77} + \frac{1}{121} = \frac{11 + 7}{847} = \frac{18}{847} \text{ in 1 hour.}$$

Then $\frac{18}{847} : 1 :: 1 \text{ hour} : x \text{ hours}$.

whence $x = \frac{847}{18}$ hours, or 47 hrs. 3 min. 20 sec. *Ans.*

7. (1) The four expressions squared become respectively

(1.) $11 + 2\sqrt{15} + 2\sqrt{10} + 2\sqrt{6} + 2\sqrt{5} + 2\sqrt{3} + 2\sqrt{2}$

(2.) $11 + 2\sqrt{15} - 2\sqrt{10} - 2\sqrt{6} - 2\sqrt{5} - 2\sqrt{3} + 2\sqrt{2}$

(3.) $11 - 2\sqrt{15} + 2\sqrt{10} - 2\sqrt{6} - 2\sqrt{5} + 2\sqrt{3} - 2\sqrt{2}$

(4.) $11 - 2\sqrt{15} - 2\sqrt{10} + 2\sqrt{6} + 2\sqrt{5} - 2\sqrt{3} - 2\sqrt{2}$

By addition we obtain 44. *Ans.*

(2) The numerator and denominator of divisor divided by $a - x$ and inverted, we have

$$\frac{a^3 - x^3}{a^3 + x^3} \times \frac{a + x}{a - x} = \frac{a^2 + a\,x + x^2}{a^2 - a\,x + x^2}. \quad \textit{Ans.}$$

8. (1) P. 60, Art. 117. (2) The series is $a\left(1 - \frac{1}{r} + \frac{1}{r^2} - \frac{1}{r^3} \ldots \textit{ad inf.}\right)$ The ratio is $-\frac{1}{r}$ $\therefore$ P. 61, VII.

$$S = \frac{a}{1 - r} = a\,.\,\frac{1}{1 + \frac{1}{r}} = \frac{a\,r}{r + 1}. \quad \textit{Ans.}$$

9. (1) P. 83, Art. 152. (2) The three quantities are—1 Principal, 2 Rate, 3 Time.

(3) P, 85, Ex. 8 (*Matric.* 1861, and B.A. 1838).

$$\begin{aligned} &\text{Let } A = m\,B\,.\,C = m^1\,C\,.\,B. \\ &\therefore A^2 = mm^1\,B^2.\,C^2 \\ &A = \sqrt{mm^1}\,B\,C \\ &\text{or } A \propto B\,.\,C. \end{aligned}$$

10. (1) $\dfrac{6\,x - 10}{x + 2} + \dfrac{40 + 8\,x}{2\,x - 3} = 10 + \dfrac{17}{6\,x^2 + 3\,x - 18}$

$$\therefore 12\,x^2 - 38\,x + 30 + 8\,x^2 + 56\,x + 80 = 20\,x^2 + 10\,x - 60 + \frac{17}{3}$$

$$\begin{aligned} \therefore 8\,x &= -170 + \frac{17}{3} \\ &= -164\tfrac{1}{3} \\ \therefore x &= -20\tfrac{13}{24}. \end{aligned}$$

(2)
$$\begin{array}{l} 12\,x + y + 3 = 15\,x - 9 \quad (1) \\ \underline{12\,y - 4\,x + 10 = 21\,y - 37 \quad (2)} \\ 3\,x - y = 12\,(1) \text{ then } 9\,y + 4\,x = 47\ (2) \\ \underline{4\,x + 9\,y = 47\ (2) \text{ and } -9\,y + 27\,x = 108\ (1)} \\ \qquad 31\,x = 155 \text{ and } x = 5. \\ \qquad \therefore y = 3. \end{array}$$

11. Let x = number of days absent from work.
Then $47 - x$ = „ „ at work. By the question

$$\begin{aligned} (47 - x) \times 2\tfrac{2}{3} - \frac{x}{2} &= 100. \\ \frac{376}{3} - \frac{8x}{3} - \frac{x}{2} &= 100 \\ 19\,x &= 152 \\ x &= 8. \quad \textit{Ans.} \end{aligned}$$

1872. *January* 10*th*.

1. (1) $\frac{133}{684}$. (2) ·3125.

2. (1) Every number is a measure of its multiple, and all common multiples of any two numbers, *other than the least*, must be *its* multiples, and therefore divisible by it. (2) 27720.

3. P. 43, Art. 91. (1) $\frac{1}{5000}$ (2) $\frac{1}{40}$. (3) $\frac{1}{220}$.

4. £4 4*s*. $1\frac{1}{2}$*d*.

5. $1\frac{142}{225}$.

6. (1) $1 - x^3 - 3x^4 - 3x^5 - x^6$.

(2) $y^4 + 2y^3 + 3y^2 + 2y + 1$.

(3) When the index is odd, $a^n + x^n$ is divisible by $a + x$. The quotient consists of terms in which the powers of *a decrease* and those of *x increase* continually, and the signs are alternately *plus* and *minus*. (See 6. (2) page 58 of this Key.)

7. (1) $\frac{1}{(1+2x)(1+3x)} - \frac{2}{(1+x)(1+3x)} + \frac{1}{(1+x)(1+2x)} =$
$\frac{(1+x)-(2+4x)+(1+3x)}{(1+x).(1+2x).(1+3x)} = \frac{2+4x-(2+4x)}{(1+x)(1+2x)(1+3x)} = 0.$ *Ans.*

(2) $x = \frac{a+b}{a-b} y = \frac{b}{a}$

$$x - y = \frac{a+b}{a-b} - \frac{b}{a} = \frac{a^2 + ab - ab + b^2}{a^2 - ab} = \frac{a^2+b^2}{a^2-ab}$$

$$xy = \frac{ab+b^2}{a^2-ab};\ 1+xy = \frac{a^2-ab+ab+b^2}{a^2-ab} = \frac{a^2+b^2}{a^2-ab} \text{ which} = x - y,$$

$$\therefore \frac{x-y}{1+xy} = 1.$$

8. (1) The right member by reduction is $\frac{3}{1-2x}$, and dividing by -1, we have $\frac{-3}{2x-1}$; then clearing of fractions

$$16x^2 - 16x + 4 - 16x^2 - 10x - 1 = -24x - 3;$$
$$2x = 6, \text{ and } x = 3. \quad \textit{Ans.}$$

$$\begin{aligned}&(2)\ (\text{I.}) \times 2\overline{a+1} = 4(a^2-1)x + 6a(a+1)y = 2a+2\\ &\quad(\text{II.}) \times a \qquad\ \ = 4a^2 \qquad x + 6a(a+1)y = 3a\end{aligned}$$

$$\therefore 4x = a - 2 \text{ and } x = \frac{a-2}{4}.$$

$$\therefore y = \frac{3-a}{6}.$$

9. (1) Art. 100.

$$(2)\ \frac{1}{1+x}=\frac{1+2x}{1+3x+2x^2}$$
$$\frac{1+x}{1+2x}=\frac{1+2x+x^2}{1+3x+2x^2}$$
$$\frac{1-x}{1}=\frac{1+2x-x^2-2x^3}{1+3x+2x^2}.$$

∴ the second is the greatest, and the third is the least.

10. (1) P. 60, Art. 117.

(2) At the 1st drawing $\frac{1}{5}$ is taken, and $\frac{4}{5}$ left,

„ 2nd „ $\frac{1}{5}$ „ $\frac{4}{5}$ of $\frac{4}{5}$ or $\frac{16}{25}$ left.

„ 5th „ $\frac{1}{5}$ of $\frac{256}{3125}$ „ $\left(\frac{4}{5}\right)^5$ or $\frac{1024}{3125}$ „

$$\therefore \frac{1}{5}+\frac{4}{5}+\frac{16}{25}+\frac{64}{225}+\frac{256}{3125}=\frac{2101}{3125}\text{ has been taken,}$$

which $=1-\frac{1024}{3125}$, as it should be.

1872. *June 26th.*

1. (1) $\frac{2}{3}$. (2) 1·84.
2. (1) £511 12*s*. 9¾*d*. (2) £6 11*s*. 3*d*.
3. (1) 10·9796229375. (2) ·002265, &c.
4. (1) 12·9505 feet. (2) 5·1802.
5. (1) P. 24, Art. 48. (2) 16170.
6. (1) $\left(\frac{a+b}{a-b}+\frac{a^2+b^2}{a^2-b^2}\right)\div\left(\frac{a-b}{a+b}-\frac{a^3-b^3}{a^3+b^3}\right)=\frac{(a+b)^2+a^2+b^2}{a^2-b^2}$

$$\div\frac{(a^2-ab+b^2)(a-b)-a^3+b^3}{a^3+b^3}=\frac{2a^2+2ab+2b^2}{a^2-b^2}\div\frac{2ab^2-2a^2b}{a^3+b^3}$$
$$=\frac{(a^2+ab+b^2)(a^2-ab+b^2)}{(a-b)(ab^2-a^2b)}=\frac{a^4+a^2b^2+b^4}{-ab(a-b)^2}=\frac{a^4+a^2b^2+b^4}{-ab^3+2a^2b^2-ab^3}.$$

$$(2)\ \frac{a}{b-\cfrac{c^2}{d-\cfrac{e^2}{x}}}=0=\frac{a}{b-\cfrac{c^2x}{dx-e^2}}=\frac{adx-ae^2}{bdx-be^x-c^2x}$$

$$\therefore adx-ae^2=0$$
$$x=\frac{ae^2}{ad}=\frac{e^2}{d}.$$

7. (1) P. 53, Art. 100. (2) If 25 : x is in duplicate ratio of 5 : 4; then x is 4^2 or 16.

(3) If $a : b :: \overline{a - c}^2 : \overline{b - c}^2$ then c is mean prop.
(note either $\overline{a - c}$ or $\overline{b - c}$ is negative).

$$\because \frac{a}{b} = \left(\frac{a - c}{b - c}\right)^2$$

$$-\frac{\surd a}{\surd b} = \frac{a - c}{b - c} \text{ or } \frac{\surd a}{\surd b} = \frac{a - c}{c - b}$$

$$\therefore c\surd a - b\surd a = a\surd b - c\surd b$$

$$\therefore c(\surd a + \surd b) = a\surd b + b\sqrt{a}$$

$$\therefore c(\surd a + \surd b) = \sqrt{a\,b}\,(\surd a + \surd b)$$

$$\therefore c = \sqrt{a\,b}. \quad \textit{Ans.}$$

8. (1) P. 58, Art. 114, III. $S = \frac{(2\,a + (n - 1)\,d)n}{2}$.

(2) Art. 114. The number of terms multiplied into the middle term gives the sum.

(3) The 60th term $14\frac{3}{17}$, is the middle term; therefore, $14\frac{3}{17} \times 119 = 1687$. *Ans.*

9. 7th term $\frac{243}{2048}$

$$S = \frac{r\,l - a}{r - 1} = \frac{\frac{3}{4} \cdot \frac{243}{2048} - \frac{2}{3}}{\frac{3}{4} - 1} = \frac{14197}{6144} = 2\,\frac{1909}{6144}. \quad \textit{Ans.}$$

$$\text{To infin., } \frac{a}{1 - r} = \frac{\frac{2}{3}}{\frac{1}{4}} = 2\tfrac{2}{3}.$$

10. (a) $\frac{3 - 2\,x}{3\left(3 - \frac{x}{2}\right)} + \frac{1}{2 - x} = 1\frac{1}{3}$

$$\frac{3 - 2\,x}{3 - \frac{x}{2}} + \frac{3}{2 - x} = 4$$

$$12 - 14\,x + 4\,x^2 + 18 - 3\,x = 48 - 32\,x + 4\,x^2$$

$$15\,x = 18$$

$$\therefore x = \frac{6}{5}. = 1\tfrac{1}{5}. \quad \textit{Ans.}$$

(b) From 2nd equation $\frac{a}{b + y} = \frac{b}{a + x}$

$$. \ a^2 + a\,x = b^2 + b\,y$$

$$\text{and } a\,x - b\,y = b^2 - a^2.$$

$$\text{Again, } a\,x + b\,y = c^2.$$

By successive addition and subtraction

$$2\,a\,x = b^2 + c^2 - a^2 \text{ and } 2\,b\,y = a^2 + c^2 - b^2.$$

$$\therefore x = \frac{b^2 + c^2 - a^2}{2\,a} \text{ and } y = \frac{a^2 + c^2 - b^2}{2\,b}.$$

11. Let x = tens' digit, y = units' digit
then $10x + y$ = the number
$10y + x =$ formed by reversing the digits.
By the terms of the question

$$10x + y + 10y + x \text{ or } x + y = 14$$
$$10x + y - 10y - x \;,, \; x - y = 4:$$
$$\therefore x = 9$$
$$y = 5$$

The number is 95.

1873. *January* 15th.

1. (1) Art. 55. (2) *g. c. m.* = 57; $\frac{111}{421}$. *Ans.*
(3) $\frac{528,\ 504,\ 320,\ 4455}{3960}$.

2. (1) $15\frac{1731}{3080}$. (2) ·00152587890625. (3) 655·36.

3. (1) From Jan. 1st, 1800, to Dec. 31st, 1863, there are 64 years, of which 15 are leap-years,

$$365 \times 64 + 15 = 23375 \text{ days}$$
$$\text{add for part of } 1864 \quad 57$$
$$23432$$
$$\text{minutes in 1 day} \quad 1440$$
$$33742080 \text{ minutes.}$$

Minutes in 1 week, 7 hours, 12 minutes are 175 × 60 + 12, or 10512 minutes,

$$\therefore \frac{10512}{33742080} \text{ or } \frac{73}{234320} \text{ is the fraction required.}$$

(2) In an acre there are 43560 square feet, $\frac{1}{43560}$ is greater than ·000022957 and less than ·000022958; so that either may be taken as the answer.

4. (1) P. 41, Art. 89. (2) $\cdot\dot{0}000\dot{1}$. (3) $\cdot\dot{0}0000999\dot{9}$.

5. (1) P. 52, Art. 99. (2) $\sqrt{\frac{3757}{65533}} = \sqrt{\frac{289}{5041}} = \frac{17}{71}$. *Ans.*

(3) 3·57771 furlongs or 28335·4534 inches, or 3 fur. 23 p. 1 ft. 9·4534. *Ans.*

6. (1) $1 - x^{16}$.

(2) It will be very useful to remember the following facts, suggested by this question, respecting the divisibility of the *sum* and *difference* of the powers of two numbers by their roots. In the general formula

$$\frac{a^n \pm x^n}{a \pm x}$$

the index n is supposed to be any whole number. The three cases are,—

I.	II.	III.
$\frac{a^2 - x^2}{a - x}$	$\frac{a^2 - x^2}{a + x}$	$\frac{a^3 + x^3}{a + x}$
$\frac{a^3 - x^3}{a - x}$	$\frac{a^4 - x^4}{a + x}$	$\frac{a^5 + x^5}{a + x}$
&c.	&c.	&c.

I. The *difference* of *all* powers is divisible by *difference* of roots.
II. „ *even powers only* „ *sum* „
III. The *sum* of *odd powers only* „ „ „
IV. „ *even* powers is not divisible by either the *sum* or *difference* of the roots.

The particular question itself is answered by the method of mathematical induction, as follows:—

$$\frac{a^n + x^n}{a + x} = a^{n-1} - \frac{a^{n-1}x - x^n}{a + x}.$$ The fraction is, $x \,.\, \frac{a^{n-1} - x^{n-1}}{a + x}$.

It is evident, therefore, that to divide $a^n + x^n$ by $a + x$ we must also divide $a^{n-1} - x^{n-1}$ by the same divisor, and if the latter is divisible so is the former; or if the division can be effected for any one power, it can also for the next higher. Now (n being supposed uneven $n - 1$ will be even) $a^2 - x^2$ is divisible by $a + x$, therefore $a^3 + x^3$ is divisible by $a + x$, and also $a^4 - x^4$, and $a^5 + x^5$, and so on; which shows that the *difference* of the even powers and the *sum* of the uneven powers of a and x are divisible by $a + x$. In a similar way the other cases may be established.

7. (1) $$\frac{bc}{\overline{a-b}\,.\,\overline{a-c}} + \frac{ca}{\overline{b-c}\,.\,\overline{b-a}} + \frac{ab}{\overline{c-a}\,.\,\overline{c-b}} =$$

$$\frac{bc(b-c) - ac(a-c) + ab(a-b)}{(a-b)(b-c)(a-c)} = \frac{b^2c - bc^2 - a^2c + ac^2 + a^2b - ab^2}{a^2b - ab^2 - a^2c + ac^2 + b^2c - bc^2}$$

$= 1.$ *Ans.*

(2) $$\frac{\frac{x-1}{x} + \frac{y-1}{y} + \frac{z+1}{z}}{\frac{1}{x} + \frac{1}{y} - \frac{1}{z}} - \frac{3xyz}{yz + xz - xy} =$$

$$\frac{xyz - yz + xyz - xz + xyz + xy - 3xyz}{yz + xz - xy} = \frac{-yz - xz + xy}{yz + xz - xy} = -1.$$ *Ans.*

8. (1) P. 53, Art. 101. (2) See question (1) June, 1870, and solution given in this Key, page 50.

9. (1) Art. 114, III.

(2) Between 10 and 1000 there will be as many numbers which when divided by 7 leave the remainder 3 as there are multiples of 7, unless 1000 happened to fall between a certain multiple of 7 and the same multiple increased by 3, which occurrence would make a difference of 1. So then between 10 and 1000 there are $\frac{1000-10}{7}$ or 141 numbers which leave the remainder 3 when divided by 7 (and 142 if 10 be included). The question then becomes—Find the sum of 141 terms of the series 17, 24, 31, &c., which $= \frac{(2\,.\,17 + 7\,.\,140)\,.\,141}{2} = 71487$, or, if 10 be included, $\frac{(2\,.\,10 + 7\,.\,141)\;142}{2}$ which $= 71497$.

10. (1) $x = 9\,a$. (2) $x = 4$. (3) $x = b + a,\ y = b - a$.

1873. *July 2nd.*

1. (1) $\dfrac{15\frac{3}{4} - \frac{14}{3} \times 1\frac{5}{6}}{\frac{1}{5} \times 23\frac{1}{3} + \frac{91}{36}} = \dfrac{\frac{63}{4} - \frac{154}{18}}{\frac{14}{3} + \frac{91}{36}} = \dfrac{\frac{567-308}{36}}{\frac{168+91}{36}} = \dfrac{259}{259} = 1.$

(2) $\dfrac{7\frac{2}{5} \times 7\frac{2}{5} \times 7\frac{2}{5} - 27}{7\frac{2}{5} \times 7\frac{2}{5} - 9} = \dfrac{7\frac{2}{5} \times 7\frac{2}{5} + 7\frac{2}{5} \times 3 + 9}{7\frac{2}{5} + 3}$

$= \dfrac{\frac{1369}{25} + 22\frac{1}{5} + 9}{\frac{52}{5}} = \dfrac{\frac{1369 + 555 + 225}{25}}{\frac{52}{5}} = \frac{2149}{260} = 8\frac{69}{260}.$

2. (1) $\dfrac{19527}{23667} = \dfrac{283}{343}$ *g. c. m.* $= 69$.

(2) Because one of the required numbers is an odd number, the common measure required must be odd. But because both the required numbers are odd, their difference is an even number, say = com. meas. × 2 : that is to say, one of the numbers is of the form np and the other $(n + 2)\,p$, where p is their greatest com. measure. Now $2\,p$ must be less than 1000: so that n must be greater than 1; and $n + 2$ must be an odd number greater than 3, say $n + 2 = 5$. And ∵ $5\,p$ must be less than 2000, put $5\,p = 1995$. This will give the two numbers 1197, and 1995; and their *g. c. m.* 399.

3. (1) $\frac{11}{512} = \cdot021484375$. (2) $\frac{7}{66} = \cdot1\dot{0}\dot{6}$.

(3) $102\cdot5 \times 1\cdot025 \times \cdot010225 = 1\cdot0742640625$.

(4) $4\cdot8 \div \cdot00016 = 30000$.

4. £2 9*s.* 7*d.* — £2 1*s.* 3*d.* = 8*s.* 4*d.* = gain on 1 quarter.

$\frac{5550}{8\frac{1}{3}}$ or $\frac{16650}{25}$ = 666 qrs. bought and sold. *Ans.*

5. (1) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{8+2\sqrt{15}}{2} = 4+\sqrt{15} = 7\cdot87298$. *Ans.*

(2) Let the length be $3\,x$ and breadth $2\,x$ $\therefore$ area is $6\,x^2$

$\therefore 6\,x^2 = 412\frac{49}{96}$ yards $= 3712\frac{19}{32}$ feet

$\therefore x^2 = 618\frac{49}{64}$

$\therefore x = \sqrt{618\frac{49}{64}} = \sqrt{\frac{39601}{64}} = \frac{199}{8} = 24\frac{7}{8}$

$\therefore$ Length $3\,x = 74\frac{5}{8}$ ft.; breadth $2\,x = 49\frac{3}{4}$ ft. *Ans.*

6. (1) $a^2 - a\,b^{\frac{2}{3}} + b^{\frac{4}{3}}$. *Ans.*

(2) $(x-1+\sqrt{2})(x-1-\sqrt{2})(x+2+\sqrt{3})(x+2-\sqrt{3})$
$= (x^3-2\,x-1)(x^2+4\,x+1) = x^4+2\,x^3-8\,x^2-6\,x-1$. *Ans.*

(3)
$$\frac{1}{4(a+\sqrt{a\,x})} + \frac{1}{2\,(a+x)} + \frac{1}{4\,(a-\sqrt{a\,x})} + \frac{x}{a^2-x^2} =$$
$$\frac{2\,a}{4\,(a^2-a\,x)} + \frac{a+x}{2\,(a^2-x^2)} = \frac{1}{2\,(a-x)} + \frac{1}{2\,(a-x)}$$
$$= \frac{1}{a-x}. \quad \textit{Ans.}$$

7. $\because a:b::c:d::e:f,$

$\therefore \frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

also $\frac{m\,a}{m\,b} = \frac{n\,c}{n\,d} = \frac{p\,e}{p\,f} = \frac{m'\,a}{m'\,b} = \frac{n'\,c}{n'\,d} = \frac{p'\,e}{p'\,f}$

$\therefore$ (see page 82) $\frac{m\,a+n\,c+p\,e}{m\,b+n\,d+p\,f} = \frac{m'\,a+n'\,c+p'\,e}{m'\,b+n'\,d+p'\,f}$

$\therefore m\,a+n\,c+p\,e : m\,b+n\,d+p\,f :: m'\,a+n'\,c+p'\,e : m'\,b+n'\,d+p'\,f.$

8. (1) The sum *ad infinitum* means that in the formulæ (p. 61, II. and VII.) $a\frac{1-r^n}{1-r}$ can be made as near as we please to $\frac{a}{1-r}$ by taking n large enough. For when r is a fraction less than unity the greater n is made the smaller r^n becomes, and by taking n large enough r^n may be neglected in comparison with unity, and then $S = a\frac{1-0}{1-r}$ or $\frac{a}{1-r}$.

In the series $1 + \frac{1}{2} + \frac{1}{4} + \ldots\ldots$ *ad inf.*

$$S = \frac{a}{1 - r} = \frac{1}{\frac{1}{2}} = 2.$$

2 is the limit to which the series constantly tends, but to which it can never be said to be exactly equal.

(2) By the question $18\,r^7 = \frac{2}{243}$ $\therefore r^7 = \frac{1}{2187}$ $\therefore r = \frac{1}{3}$

$\therefore$ 5th term is $18 \times \left(\frac{1}{3}\right)^4 = \frac{2}{9}$.

"Sum of that," &c. $= \frac{2}{9} \cdot \frac{1}{1 - \frac{1}{3}} = \frac{1}{3}$. *Ans.*

9. (1) $\frac{2\,(4\,x - 3)}{x - 3} - \frac{3}{x - 1} = 8.$

$\therefore 2\,(4\,x - 3)\,(x - 1) - 3\,(x - 3) = 8\,(x - 3)\,(x - 1)$

$8\,x^2 - 14\,x + 6 - 3\,x + 9 = 8\,x^2 - 32\,x + 24$

$15\,x = 9 \quad \therefore x = \frac{3}{5}$. *Ans.*

(2) $\frac{\sqrt{x + 12} + \sqrt{12}}{\sqrt{x + 12} - \sqrt{12}} = 2 \;\therefore \sqrt{x + 12} + \sqrt{12} =$

$2\sqrt{x + 12} - 2\sqrt{12}$, and $\sqrt{x + 12} = 3\sqrt{12}$

$\therefore x + 12 = 108$ and $x = 96$. *Ans.*

(3) (ii.) − (i.) . . $5\,z - 3\,y = 26 \ldots\ldots$ (iv.)

(i.) + (iii.) $4\,y - z = 22 \ldots\ldots$ (v.)

(v.) × 5 + (iv.) $17\,y = 136$

$$\left.\begin{aligned} \therefore y &= 8 \\ z &= 10 \\ x &= 6 \end{aligned}\right\} \textit{ Ans.}$$

10. Say x gallons of the former and y of the latter.

x gallons of the 1st mixture contain $\frac{5\,x}{12}$ water, and $\frac{7\,x}{12}$ wine

y „ 2nd „ $\frac{8\,y}{19}$ water, and $\frac{11\,y}{19}$ wine

$$\therefore \frac{5\,x}{12} + \frac{8\,y}{19} = h \ldots\ldots (1)$$

$$\frac{7\,x}{12} + \frac{11\,y}{19} = k \ldots (2)$$

$$(1) \times 11 \ldots \frac{55\,x}{12} + \frac{88\,y}{19} = 11\,h$$

$$(2) \times 8 \ldots \frac{56\,x}{12} + \frac{88\,y}{19} = 8\,k$$

$$\left.\begin{aligned} &\therefore \frac{x}{12} = 8\,k - 11\,h \;\therefore x = 96\,k - 132\,h \\ &\text{Similarly } \frac{y}{19} = 7\,h - 5\,k \;\therefore y = 133\,h - 95\,k \end{aligned}\right\} \textit{ Ans.}$$

1874. *January* 14*th*.

1. Art. 42, p. 21. (2) 399. (3) 65520.

2. $\frac{11539}{50625}$. (2) £4 7*s*. 2¼*d*.

3. ·38048. (2) 156250. (3) $\cdot 0004236\dot{1}$.

4. Length 246·9, breadth 123·45 feet.

5. $101{\cdot}75 : 100 :: 1356{\cdot}\dot{6} : 1333{\cdot}\dot{3}$
and £1356 13*s*. 4*d*. – £1333 6*s*. 8*d*. = £23 6*s*. 8*d*.
(2) £1356 13*s*. 4*d*. × 5¼ ÷ 100 = £71 4*s*. 6*d*.

6. $1 - \left(a^2 - \frac{1}{a^2}\right) x^2 + x^4.$

(2)

$$\begin{array}{l}
x^2 + x\sqrt{2}+1 \,\big)\, x^6 + \quad x^4 \quad + x^3 + 1 \,\big(\, x^4 - x^3\sqrt{2} + 2x^2 + (1-\sqrt{2})x - \sqrt{2} \\
\hspace{22em} + \frac{1+\sqrt{2}}{x} - \frac{1}{x^2} \\
\qquad\qquad\quad \underline{x^6 + x^5\sqrt{2} + x^4} \\
\qquad\qquad\quad -x^5\sqrt{2} + x^3 + 1 \\
\qquad\qquad\quad \underline{-x^5\sqrt{2} - 2x^4 - x^3\sqrt{2}} \\
\qquad\qquad\qquad 2x^4 + (1+\sqrt{2})x^3 + 1 \\
\qquad\qquad\qquad \underline{2x^4 + \ (2\sqrt{2})\ x^3 + 2x^2} \\
\qquad\qquad\qquad\quad (1-\sqrt{2})x^3 - 2x^2 + 1 \\
\qquad\qquad\qquad\quad \underline{(1-\sqrt{2})x^3 + (\sqrt{2}-2)x^2 + (1-\sqrt{2})x} \\
\qquad\qquad\qquad\qquad -x^2\sqrt{2} + (\sqrt{2}-1)x + 1 \\
\qquad\qquad\qquad\qquad \underline{-x^2\sqrt{2} - \quad 2x \quad - \sqrt{2}} \\
\qquad\qquad\qquad\qquad\quad (1+\sqrt{2})x + 1 + \sqrt{2} \quad \{=(x+1)(1+\sqrt{2})\} \\
\qquad\qquad\qquad\qquad\quad \underline{(1+\sqrt{2})x + 2 + \sqrt{2} + \frac{1+\sqrt{2}}{x}} \\
\qquad\qquad\qquad\qquad\qquad\quad -1 - \frac{1+\sqrt{2}}{x}.
\end{array}$$

(3) $(x^2 + a - 1 + a\,x)\ (x^2 + a - 1\,a\,x)$
or $\{x(x + a) + a - 1\}\ \{x\,(x - a) + a - 1\}$.

7. (*α*) $\frac{5}{3}(a^2 + a\,b + b^2)$.

(*β*) G. C. M. $x - 1$; $-\frac{1 + 3\,x}{1 + 2\,x + 4\,x^2}$.

(*γ*) $$\frac{a^2\,x^2 + b^2\,x^2 + a^2\,y^2 + b^2\,y^2}{\frac{a^2}{x^2} + \frac{b^2}{x^2} + \frac{a^2}{y^2} + \frac{b^2}{y^2}} = \frac{(a^2 + b^2)\,(x^2 + y^2)}{(a^2 + b^2)\left(\frac{1}{x^2} + \frac{1}{y^2}\right)}$$

$$= \frac{x^2 + y^2}{\frac{1}{x^2} + \frac{1}{y^2}} = x^2\,y^2.$$

8. Arts. 103 and 107.

(2) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a^2}{b^2} = \frac{c^2}{d^2}$, and $\frac{m\,a^2}{n\,b^2} = \frac{m\,c^2}{n\,d^2}$

$$\therefore \frac{m\,a^2}{n\,b^2} \pm 1 = \frac{m\,c^2}{n\,d^2} \pm 1\,; \text{ or } \frac{m\,a^2 \pm n\,b^2}{n\,b^2} = \frac{m\,c^2 + n\,d^2}{n\,d^2}.$$

$$\text{But } \frac{m\,a^2 + n\,b^2}{n\,b^2} \div \frac{m\,a^2 - n\,b^2}{n\,b^2} = \frac{m\,c^2 + n\,d^2}{n\,d^2} \div \frac{m\,c^2 - n\,d^2}{n\,d^2}.$$

$$\text{that is } \frac{m\,a^2 + n\,b^2}{m\,a^2 - n\,b^2} = \frac{m\,c^2 + n\,d^2}{m\,c^2 - n\,d^2}$$

$$\text{or } m\,a^2 + n\,b^2 : m\,a^2 - n\,b^2 :: m\,c^2 + n\,d^2 : m\,c^2 + n\,d^2.$$

(3) Art. 148.

9. (α) $x^2 - (m + n)\,b\,x + m\,n\,b^2 = -\,(m + n)\,a\,x + m\,n\,a^2.$

$$m\,n\,a^2 - m\,n\,b^2 = (m + n)\,a\,x - (m + n)\,b\,x$$

$$m\,n\,(a^2 - b^2) = (m + n)\,(a - b)\,x$$

$$x = \frac{m\,n\,(a^2 - b^2)}{(m + n)\,(a - b)}$$

$$= \frac{m\,n}{m + n}\,(a + b).$$

(β) $x = -\,9.$

(γ) $x = \frac{b^2}{a - b}\,;\ y = \frac{a^2}{a - b}.$

10. $\frac{(50 + 99 \times -\frac{1}{2})\,100}{2} = 50.$

(2) The same three numbers whether equal or not cannot be at once in Arithmetical and Geometrical Progression. See Arts. 112, 116, 189, also Qu. 6 (2) p. xxi. B.A. Exam. 1859, and the answer in Key.

1874. *July* 1*st.*

1. $34\cdot\dot{6}.$
2. 66.
3. $\cdot 0\dot{1}707\dot{3}.$

(2) $\frac{6}{11{,}111}.$

(3) A vulgar fraction in its lowest terms whose denominator contains only the powers of 2 and 5 as factors will always reduce to a non-recurrent decimal (Art. 88).

(4) Yes; see Art. 88.

4. $\frac{123}{427}$.

(2) $\sqrt{11\frac{37}{49}} = \sqrt{11 \cdot 7551020481, \&c.} = 3 \cdot 42857$, &c.

(3) $\sqrt{76 \cdot 8235294117, \&c.} = 8 \cdot 76490$, &c.

5. Arts. 41 and 42.

6. $\frac{2\,x^3 + x}{x^4 - 1}$; (2) $\frac{2\,a^3 + a\,b^2}{a^4 - b^4}$.

7. The ratios added are,

$$\frac{a^3 - b^3}{a^2 - b^2} + \frac{a^3 + b^3}{a^2 - b^2} = \frac{2\,a^3}{a^2 - b^2};$$

$$\left.\begin{array}{l} \text{then } \frac{2\,a^3}{a^2 - b^2} : \frac{a^3 - b^3}{a^2 - b^2} :: 2\,a^3 : a^3 - b^3 \\ \frac{2\,a^3}{a^2 - b^2} : \frac{a^3 + b^3}{a^2 - b^2} :: 2\,a^3 : a^3 + b^3 \end{array}\right\} \textit{Ans.}$$

8. Art. 114, III.

$$s = \frac{(2\,a + (\overline{n - 1\,d})\,n)}{2}\;\therefore\; d = \frac{2\,(s - a\,n)}{n\,(n - 1)}.$$

(2) Art. 158, IV.

$$r = \frac{100\,\text{I}}{P\,t}, \text{ or } \frac{100 \times 8700}{800 \times 30} = 36\tfrac{1}{4}.$$

9. a = the difference of the two magnitudes sought for.
b = their sum.

$\therefore \frac{b - a}{2}$ = the less, or the qth term of the series,

and $\frac{b - a}{2} + a$ or $\frac{b + a}{2}$ = the pth term.

10. (1) $x = \frac{7}{8}$ (2) $x = \frac{1}{6}, y = -\frac{1}{3}, z = 1$.

11. Let x = the number of guineas, then by question,

$(4\,x - 17)\,20 = 2100 - 42\,x$; and solving,

$x = 20$ = the number of guineas.

$\therefore$ $(80 - 17)$ or 63 = the number of half-sovereigns.

1875. *January* 13*th*.

1. 138 days.
2. (1) Page 24, Art. 47. (2) 294840.
3. $\frac{152}{299}$ of 897*s*. = £22 16*s*.
4. (1) $\cdot 0000134150\dot{4}\dot{0}$. (2) $\epsilon - 2 = \cdot 71828$, &c.

5. £37 17*s.* $3\frac{1}{10}$*d.*

6. $\frac{7-\sqrt{11}}{7+2\sqrt{11}} = \frac{71-21\sqrt{11}}{5} = \cdot 27028.$

$$(2)\ \sqrt{21} - \sqrt{18} = \frac{2\sqrt{42}-12}{8}.$$

Now $\sqrt{42}$ is less than 7, therefore either member of the above equation is less than $\frac{2}{8}$, and *à fortiori*, is less than $\frac{3}{8}$.

(3) $\sqrt{3} + \sqrt{7}$ is less than $\sqrt{3} + \sqrt{3} + 2$. First, removing $\sqrt{3}$ from each, which does not alter their difference, $\sqrt{7}$ is less than $\sqrt{3} + 2$, because the former is *less*, and the latter *greater*, than 3.

7. (1) G.C.M. $x^2 - 3x - 4$; $\frac{x^2 - 6x + 2}{x^2 - 5x + 3}$. (2) $\frac{1}{x^2}$.

8. Let x and y = the Nos.; $x : y :: 10 : 8$, and

$$\left(x - \frac{34}{3}\right) : \left(y - \frac{34}{3}\right) :: 13 : 10,$$

$$\therefore \begin{cases} x = 85. \\ y = 68. \end{cases}$$

9. (1) $s = 10\frac{(\frac{11}{10})^{10}-1}{\frac{11}{10}-1} = 10\frac{2\cdot 5937424601-1}{\frac{1}{10}} = 159\cdot 37424601.$

(2) 768 stones $2\frac{1}{2}$ feet apart, same as 256, $7\frac{1}{2}$ feet apart.

$$s = 35 + 50 + 65 + \text{\&c. to 255 terms.}$$

$$\text{Last term} = 35 + \overline{15 \times 255} = 3860.$$

$$\therefore s = \frac{35 + 3860}{2} \,.\, 256 = 498560 \text{ feet.}$$

10. (1) $x = \frac{b-a}{2a-1}$; $y = \frac{a+b}{2a-1}$.

(2) $x = 6$; $y = 12$; $z = 60$.

11. a is either 0 or 1.

If $a = 0$ then the sum of even numbers would be an odd number, *i.e.* 67.

$\therefore a$ is not $= 0$, and $a = 1$.

But $b + 2c + 4d + \ldots = 33$, and $b = 0$ or 1.

If $b = 0$ then the sum &c. (as before) $\therefore b$ also $= 1$.

$\therefore c + 2d + 4e + \ldots = 16$

and $c = 0$ on the sum &c. $\therefore d + 2e + 4f + \ldots = 8$

$\therefore d$ also $= 0$, and $e + 2f + 4g + \ldots\ldots = 4$

$\therefore e = 0$, and $f + 2g + 4h + \ldots\ldots = 2$

$\therefore f = 0$, and $g + 2h +$ &c. $\ldots\ldots = 1$

$\therefore g = 1$, and all that follow $= 0$

Wherefore $a = 1$, $b = 1$, and $g = 1$. All the rest $= 0$.

1875. *June 30th.*

1. (1) $\frac{102}{133}$.

   (2) $\frac{33124 + 14560 + 24336 + 23205 + 15210}{70980} = 1\frac{607}{1092}$.

2. (1) $\frac{232}{129} = 1\frac{103}{129}$; (2) $\frac{172}{271}$. (3) $\frac{5547}{15718}$.

3. (1) 39·3708 English inches = 38·2344 Prussian inches,

   ∴ the ratio of the English to the Prussian is $\frac{382344}{393708}$

and the Prussian exceeds the English by

$$\frac{393708 - 382344}{393708} = \frac{2841}{98417}.$$

(2) As $(3{\cdot}28)^2 : (3{\cdot}18)^2 :: 9 : x$

or $10{\cdot}7584 : 10{\cdot}1124 :: 9 : 8{\cdot}459$ Prussian sq. feet.

4. $\frac{33}{67}$.

(2) "A ratio of greater inequality is diminished, and a ratio of less inequality is increased, by an equal increase of the numerator and denominator."

5. (1) 79. (2) 2·1646.
6. (1) By 13^2. (2) By 13^2.
7. (1) $x^4 - 2\,a\,x^2 + a^2 - 2\,b^2\,x^2 - 2\,a\,b^2 + b^4$.

   (2) $8\,a^2 - 2\,a\,b - 10\,a\,c - 3\,b^2 + 5\,b\,c + 2\,c^2$.
8. (1) 1. (2) $\frac{2\,x\,y}{x^2 + y^2}$.
9. (1) $\frac{895}{5} = 179$. (2) $\frac{(210 + 178 \times 5)\,179}{2} = 98450$.
10. (1) $x = 15$. (2) $x = 44$.
11. $x = 34$, $y = 27$, $z = 42$. *Ans.*
12. 1st tube fills $\frac{1}{6}$th in 1 hour

    2nd „ $\frac{1}{8}$th do.

    3rd empties $\frac{1}{4}$th do.

The three tubes in one hour fill $\frac{1}{6} + \frac{1}{8} - \frac{1}{4} = \frac{1}{24}$th, and in 12 hours the cistern will be filled, as it is $\frac{1}{2}$ full to begin with.

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